Homework 2 (by Prashasthi Prabhakar)
Due: Wednesday Sep 20, 2017 @ 11:59pm

IMPORTANT:
• Upload a PDF of your answers to Gradescope by 11:59pm on Wednesday Sep 20, 2017.
• Plagiarism: Homework may be discussed with other students, but all homework is to be completed individually.
• Typeset all your answers.

For your information:
• Graded out of 100 points; 4 questions total
• Rough time estimate: ≈1-4 hours (0.5-1 hours for each question)

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Question 1: Functional Dependencies I ........................ [15 points]

Consider the following legal instance of a relational schema $S$ with attributes $X Y Z$:

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Table 1: Legal instance of schema $S$ for question 2.1

(a) Which of the following dependencies are violated by the instances of $S$ in Table 1?

i. [2 points] □ Yes □ No : $X \rightarrow Y$ is violated.

ii. [2 points] □ Yes □ No : $Z \rightarrow X$ is violated.

iii. [2 points] □ Yes □ No : $Y \rightarrow Z$ is violated.

iv. [2 points] □ Yes □ No : $XY \rightarrow Z$ is violated.

v. [2 points] □ Yes □ No : $YZ \rightarrow X$ is violated.

vi. [2 points] □ Yes □ No : $XZ \rightarrow Y$ is violated.

(b) [3 points] By only observing the instance of $S$ in Table 1, can you identify the functional dependencies that hold on schema $S$? Why?

□ Yes □ No
Question 2: Functional Dependencies II .......................... [32 points]

For the next set of questions consider the relational schema \( R = \{ P, Q, R, S, T, U, V, W \} \) and the set of functional dependencies FD:

\[
\begin{align*}
Q & \rightarrow U & (1) \\
U & \rightarrow V & (2) \\
PQ & \rightarrow WST & (3) \\
SU & \rightarrow TR & (4) \\
VT & \rightarrow RW & (5) \\
R & \rightarrow W & (6)
\end{align*}
\]

(a) [8 points] Which of the following is a minimum cover of the FD? Mark all that qualify; if none, mark accordingly, and give your own answer.

i. The given FDs (Eq 1-6), is a minimum cover already.
ii. \{Q \rightarrow U, U \rightarrow V, PQ \rightarrow S, SU \rightarrow T, SU \rightarrow R, VT \rightarrow R, VT \rightarrow W, R \rightarrow W\}
iii. \{Q \rightarrow U, U \rightarrow V, PQ \rightarrow S, SU \rightarrow T, PQ \rightarrow W, VT \rightarrow R, PQ \rightarrow T, R \rightarrow W\}
iv. \{Q \rightarrow U, U \rightarrow V, PQ \rightarrow S, SU \rightarrow T, VT \rightarrow R, R \rightarrow W\}
v. \{Q \rightarrow U, U \rightarrow V, PQ \rightarrow S, SU \rightarrow T, SU \rightarrow R, VT \rightarrow R, PQ \rightarrow T, R \rightarrow W\}
vii. none of the above - the cover is

(b) Yes/No: Which of the following functional dependencies can be deduced, from the above set of functional dependencies (Eq. (1)-(6))?  

i. [3 points] □ Yes □ No : Q \rightarrow V
ii. [3 points] □ Yes □ No : QU \rightarrow R
iii. [3 points] □ Yes □ No : SQ \rightarrow T
iv. [3 points] □ Yes □ No : SQ \rightarrow W
v. [3 points] □ Yes □ No : PQ \rightarrow R
vi. [3 points] □ Yes □ No : VT \rightarrow Q

(c) [3 points] True or False: The attribute closure \( \{Q\}^+ \) is \( \{Q, U, V\} \).
□ True □ False

(d) [3 points] True or False: The attribute closure \( \{PQ\}^+ \) is \( \{P, Q, W, S, T\} \).
□ True □ False

Homework 2 continues...
Question 3: Decompositions............................................[20 points]

For this set of questions, consider the relation with attributes, \( \mathcal{X} = \{A, B, C, D, E, F\} \), Let the following functional dependencies \( FD \) be defined over the relation \( \mathcal{X} \):

\[
A \rightarrow B \\
B \rightarrow CD \\
E \rightarrow F
\]

(a) [2 points] Provide the attribute closure of \( \{AB\} \).

(b) Consider the decomposition \( AB, BCD, EF \). Mark 'True' or 'False':
   i. [3 points] \( \Box \) True \( \Box \) False : It is lossless
   ii. [3 points] \( \Box \) True \( \Box \) False : It is dependency-preserving

(c) Consider the decomposition \( AB, BCDF, EF \). Mark 'True' or 'False':
   i. [3 points] \( \Box \) True \( \Box \) False : It is lossless
   ii. [3 points] \( \Box \) True \( \Box \) False : It is dependency-preserving

(d) Consider the decomposition \( ABCF, EBD \). Mark 'True' or 'False':
   i. [3 points] \( \Box \) True \( \Box \) False : It is lossless
   ii. [3 points] \( \Box \) True \( \Box \) False : It is dependency-preserving

Homework 2 continues...
Question 4: Normal Forms.............................................[33 points]
Consider the relation with attributes, \( E = \{ P, Q, R, S \} \). Suppose that the following functional dependencies hold:

\[
\begin{align*}
PQ & \rightarrow R \\
PQ & \rightarrow S \\
R & \rightarrow P \\
S & \rightarrow Q
\end{align*}
\]

(a) [6 points] List all the candidate key(s) for \( E \).

(b) [2 points] Is the relation \( E \) in BCNF? □ Yes □ No

(c) From the list below, select all applicable choices to justify whether \( E \) is (or is not) in BCNF.

Note: when we refer to the main requirement for BCNF, we mean: every determinant is a super key.

i. [1 point] □ True □ False: All FD’s satisfy the main requirement.

ii. [1 point] □ True □ False: FD (7) violates the main requirement.

iii. [1 point] □ True □ False: FD (8) violates the main requirement.

iv. [1 point] □ True □ False: FD (9) violates the main requirement.

v. [1 point] □ True □ False: FD (10) violates the main requirement.

(d) [2 points] Is the relation \( E \) in 3NF? □ Yes □ No

(e) From the list below, select all applicable choices to justify whether \( E \) is (or is not) in 3NF.

Note: when we refer to the secondary requirement for 3NF, we mean: for every FD \( X \rightarrow A \), \( A \) is part of a candidate key.

i. [1 point] □ True □ False: All FD’s satisfy the secondary requirement.

ii. [1 point] □ True □ False: FD (7) violates the secondary requirement.

iii. [1 point] □ True □ False: FD (8) violates the secondary requirement.

iv. [1 point] □ True □ False: FD (9) violates the secondary requirement.

v. [1 point] □ True □ False: FD (10) violates the secondary requirement.

(f) [5 points] Give a 3NF decomposition of \( E \) that is lossless, dependency preserving, and has as few tables as possible.

(g) [8 points] Give a BCNF decomposition of \( E \) that is lossless, and has as few tables as possible.