IMPORTANT:

- **Upload a PDF** of your answers to Gradescope by **11:59pm on Wednesday Sep 20, 2017**.
- **Plagiarism**: Homework may be discussed with other students, but all homework is to be completed **individually**.
- **Typeset** all your answers.

For your information:

- Graded out of **100** points; **4** questions total
- Rough time estimate: ≈1-4 hours (0.5-1 hours for each question)

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Question 1: Functional Dependencies I . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . [15 points]

GRADED BY: Mengran

Consider the following legal instance of a relational schema \( S \) with attributes \( XYZ \):

\[
\begin{array}{c|c|c}
S & X & Y \\
\hline
m & 20 & T \\
m & 10 & F \\
o & 30 & T \\
n & 30 & T \\
o & 20 & T \\
\end{array}
\]

Table 1: Legal instance of schema \( S \) for question 2.1

(a) Which of the following dependencies are violated by the instances of \( S \) in Table 1?

i. [2 points] ☐ Yes  ☑ No: \( X \rightarrow Y \) is violated.

ii. [2 points] ☐ Yes  ☑ No: \( Z \rightarrow X \) is violated.

iii. [2 points] ☐ Yes  ☑ No: \( Y \rightarrow Z \) is violated.

iv. [2 points] ☐ Yes  ☑ No: \( XY \rightarrow Z \) is violated.

v. [2 points] ☐ Yes  ☑ No: \( YZ \rightarrow X \) is violated.

vi. [2 points] ☐ Yes  ☑ No: \( XZ \rightarrow Y \) is violated.

(b) [3 points] By only observing the instance of \( S \) in Table 1, can you identify the functional dependencies that hold on schema \( S \)? Why?

☐ Yes  ☑ No

Solution: No, because we can only see an instance.
Question 2: Functional Dependencies II ......................... [32 points]
GRADED BY: Allison

For the next set of questions consider the relational schema \( \mathcal{R} = \{P, Q, R, S, T, U, V, W\} \) and the set of functional dependencies FD:

\[
\begin{align*}
Q & \rightarrow U \\
U & \rightarrow V \\
PQ & \rightarrow WST \\
SU & \rightarrow TR \\
VT & \rightarrow RW \\
R & \rightarrow W
\end{align*}
\]

(a) [8 points] Which of the following is a minimum cover of the FD? Mark all that qualify; if none, mark accordingly, and give your own answer.

i. The given FDs (Eq 1-6), is a minimum cover already.
ii. \( \{Q \rightarrow U, U \rightarrow V, PQ \rightarrow S, SU \rightarrow T, SU \rightarrow R, VT \rightarrow R, VT \rightarrow W, R \rightarrow W\} \)
iii. \( \{Q \rightarrow U, U \rightarrow V, PQ \rightarrow S, SU \rightarrow T, PQ \rightarrow W, VT \rightarrow R, PQ \rightarrow T, R \rightarrow W\} \)
iv. \( \{Q \rightarrow U, U \rightarrow V, PQ \rightarrow S, SU \rightarrow T, VT \rightarrow R, R \rightarrow W\} \)
v. \( \{Q \rightarrow U, U \rightarrow V, PQ \rightarrow S, SU \rightarrow R, VT \rightarrow R, PQ \rightarrow T, R \rightarrow W\} \)
vi. none of the above - the cover is ____________

Solution: iv

(b) Yes/No: Which of the following functional dependencies can be deduced, from the above set of functional dependencies (Eq. (1)-(6))? 

i. [3 points] ■ Yes □ No : \( Q \rightarrow V \)
ii. [3 points] □ Yes ■ No : \( QU \rightarrow R \)
iii. [3 points] ■ Yes □ No : \( SQ \rightarrow T \)
iv. [3 points] ■ Yes □ No : \( SQ \rightarrow W \)
v. [3 points] ■ Yes □ No : \( PQ \rightarrow R \)
vi. [3 points] □ Yes ■ No : \( VT \rightarrow Q \)

(c) [3 points] True or False: The attribute closure \( \{Q\}^+ \) is \( \{Q, U, V\} \).

■ True □ False

(d) [3 points] True or False: The attribute closure \( \{PQ\}^+ \) is \( \{P, Q, W, S, T\} \).

□ True ■ False

Grading info: It is \( \{P, Q, R, S, T, U, V, W\} \).

Homework 2 continues...
Question 3: Decompositions........................................[20 points]
GRADING BY: Prashasthi

For this set of questions, consider the relation with attributes, \( \mathcal{X} = \{A, B, C, D, E, F\} \), Let the following functional dependencies \( FD \) be defined over the relation \( \mathcal{X} \):

\[
\begin{align*}
A & \rightarrow B \\
B & \rightarrow CD \\
E & \rightarrow F
\end{align*}
\]

(a) [2 points] Provide the attribute closure of \( \{AB\} \).

Solution: \( \{AB\}^+ = \{ABCD\} \)

(b) Consider the decomposition \( AB, BCD, EF \). Mark 'True' or 'False':
   i. [3 points] □ True  ■ False : It is lossless
   ii. [3 points] ■ True  □ False : It is dependency-preserving

(c) Consider the decomposition \( AB, BCDF, EF \). Mark 'True' or 'False':
   i. [3 points] □ True  ■ False : It is lossless
   ii. [3 points] ■ True  □ False : It is dependency-preserving

(d) Consider the decomposition \( ABCEF, EBD \). Mark 'True' or 'False':
   i. [3 points] ■ True  □ False : It is lossless
   ii. [3 points] ■ True  □ False : It is dependency-preserving

Homework 2 continues...
Question 4: Normal Forms.................................[33 points]
GRADED BY: Leon

Consider the relation with attributes, \( E = \{P, Q, R, S\} \). Suppose that the following functional dependencies hold:

\[
\begin{align*}
PQ & \rightarrow R \quad (7) \\
PQ & \rightarrow S \quad (8) \\
R & \rightarrow P \quad (9) \\
S & \rightarrow Q \quad (10)
\end{align*}
\]

(a) [6 points] List all the candidate key(s) for \( E \).

**Solution:** \{PQ\}, \{QR\}, \{RS\}, \{PS\}

*Grading info:* -2: for each missing candidate key

(b) [2 points] Is the relation \( E \) in BCNF? □ Yes ■ No

(c) From the list below, select all applicable choices to justify whether \( E \) is (or is not) in BCNF.

**Note:** when we refer to the main requirement for BCNF, we mean: *every determinant is a super key.*

i. [1 point] □ True ■ False : All FD’s satisfy the main requirement.

ii. [1 point] □ True ■ False : FD (7) violates the main requirement.

iii. [1 point] □ True ■ False : FD (8) violates the main requirement.

iv. [1 point] ■ True □ False : FD (9) violates the main requirement.

v. [1 point] ■ True □ False : FD (10) violates the main requirement.

(d) [2 points] Is the relation \( E \) in 3NF? ■ Yes □ No

(e) From the list below, select all applicable choices to justify whether \( E \) is (or is not) in 3NF.

**Note:** when we refer to the secondary requirement for 3NF, we mean: *for every FD \( X \rightarrow A \), \( A \) is part of a candidate key.*

i. [1 point] ■ True □ False : All FD’s satisfy the secondary requirement.

ii. [1 point] □ True ■ False : FD (7) violates the secondary requirement.

iii. [1 point] □ True ■ False : FD (8) violates the secondary requirement.

iv. [1 point] □ True ■ False : FD (9) violates the secondary requirement.

v. [1 point] □ True ■ False : FD (10) violates the secondary requirement.

(f) [5 points] Give a 3NF decomposition of \( E \) that is lossless, dependency preserving, and has as few tables as possible.

**Solution:** \( E_{1,1} = \{P, Q, R, S\} \)

*Grading info:* -1: 3NF decomposition with 2 tables; -3: 3NF decomposition more than with 2 tables; -4: 3NF decomposition with more than 5 tables

Question 4 continues...
(g) [8 points] Give a BCNF decomposition of $E$ that is lossless, and has as few tables as possible.

**Solution:** $E_{1,1} = (P, R), E_{1,2} = (Q, S), E_{1,3} = (R, S)$

*Grading info:* -5: BCNF decomposition with more than 3 tables