

CARNEGIE MELLON UNIVERSITY  
DEPARTMENT OF COMPUTER SCIENCE  
15-445/645 – DATABASE SYSTEMS (FALL 2017)  
PROF. ANDY PAVLO

Homework 4 (by Sivaprasad Sudhir) – Solutions  
Due: **Wednesday Oct 11, 2017 @ 11:59pm**

**IMPORTANT:**

- **Upload this PDF** with your answers to **Gradescope by 11:59pm on Wednesday Oct 11, 2017.**
- **Plagiarism:** Homework may be discussed with other students, but all homework is to be completed **individually.**
- **You have to use this PDF for all of your answers.**

For your information:

- Graded out of **100** points; **2** questions total
- Rough time estimate:  $\approx$  1 - 2 hours (0.5 - 1 hours for each question)

*Revision : 2017/10/15 21:44*

Question	Points	Score
Sorting	40	
Join Algorithms	60	
Total:	100	

**Question 1: Sorting.....[40 points]****Graded by:**

We have a file with a million pages ( $N = 1,000,000$  pages), and we want to sort it using external merge sort. Assume the simplest algorithm, that is, no double buffering, no blocked I/O, and quicksort for in-memory sorting. Let  $B$  denote the number of buffers.

(a) **[10 points]** What is the smallest number of buffers  $B$ , that can sort the file with  $N = 1,000,000$  pages, in 2 passes?

- 32    33    34    99    100    101    102    999    1,000  
 **1,001**    1,000,000    1,000,001

**Solution:** We want  $B$  where  $N \leq B * (B - 1)$  If  $B = 1001$ , then  $1,000,000 \leq 1001 * 1000 = 1,001,000$ ; smaller  $B$ , fails.

(b) **[10 points]** What is the smallest number of buffers  $B$ , that can sort the file with  $N = 1,000,000$  pages, in 3 passes?

- 32    33    34    99    100    **101**    102    999    1,000  
 1,001    1,000,000    1,000,001

**Solution:**  $B * (B - 1)^2 = 101 * 100 * 100 = 1,010,000$ . Anything less, fails.

(c) **[10 points]** How many passes are needed to sort the file with  $N = 1,000,000$  pages with 6 buffers?

- 7    8    **9**    10    11

**Solution:**  $1 + \text{ceil}(\log_{B-1}(\text{ceil}(N/B))) = 1 + \text{ceil}(\log_5(166,667)) = 9$

(d) **[10 points]** What is the total I/O cost to sort the file with  $N = 1,000,000$  pages with 6 buffers?

- 14,000,000    8,000,000    **18,000,000**    10,000,000    22,000,000

**Solution:**  $Cost = 2 * N * \#passes = 2 * 1,000,000 * 9$

**Question 2: Join Algorithms ..... [60 points]****Graded by:**

Consider relations  $R(x, y)$  and  $S(x, z)$  to be joined on the common attribute  $x$ . Assume that there are no indexes.

- There are  $B = 30$  pages in the buffer
- Table  $R$  spans  $M = 1800$  pages with 50 tuples per page
- Table  $S$  spans  $N = 500$  pages with 100 tuples per page

What are the I/O costs for the following joins?

- Assume the simplest cost model, where pages are read and written one at a time
- Assume that you will need one buffer block to hold the evolving output block and one input block to hold the current input block of the inner relation
- Ignore the cost of the final writing of the results

- (a) **[10 points]** Block nested loop join with  $R$  as the outer relation and  $S$  as the inner relation  
 30,000    31,800    32,900    33,300    **34,300**

**Solution:**  $M + \text{ceil}(M / (B - 2)) * N = 1800 + \text{ceil}(1800 / 28) * 500 = 34,300$

- (b) **[5 points]** Block nested loop join with  $S$  as the outer relation and  $R$  as the inner relation  
 29,300    31,100    31,800    **32,900**    34,300

**Solution:**  $N + \text{ceil}(N / (B - 2)) * M = 500 + \text{ceil}(500 / 28) * 1800 = 32,900$

- (c) Hash join with  $S$  as the outer relation and  $R$  as the inner relation (Ignore recursive partitioning and partially filled blocks)

- i. **[5 points]** Cost of partition phase  
 2,300    **4,600**    6,900    3,600    1,000

**Solution:**  $2 * (M + N) = 2 * (1800 + 500) = 4600$

- ii. **[5 points]** Cost of probing phase  
 **2,300**    4,600    6,900    3,600    1,000

**Solution:**  $(M + N) = (1800 + 500) = 2300$

- (d) Sort-merge join

- i. **[10 points]** Cost of sorting  $R$   
 8966    **7934**    6578    1828    2204

**Solution:**  $2 * M * \log(M) / \log(B) = 2 * 1800 * \log(1800) / \log(30) = 7934$

- ii. **[5 points]** Cost of sorting  $S$   
 8966    7934    6578    **1828**    2204

**Solution:**  $2 * N * \log(N) / \log(B) = 2 * 500 * \log(500) / \log(30) = 1828$

iii. **[10 points]** Cost of merge assuming no duplicates in the join attribute

**2,300**    4,600    6,900    154    77

**Solution:**  $M + N = 2300$

iv. **[10 points]** Cost of merge in the worst case

6,900    31,800    33,300    **900,000**    1,800,000

**Solution:**  $M * N = 900000$