1 Sorting

We need sorting because in the relation model, tuples in a table have no specific order. Sorting is (potentially) used in ORDER BY, GROUP BY, JOIN, and DISTINCT operators.

We can accelerate sorting using a clustered B+tree by scanning the leaf nodes from left to right. This is a bad idea, however, if we use an unclustered B+tree to sort because it causes a lot of I/O reads (random access through pointer chasing).

If the data that we need to sort fits in memory, then we can use standard sorting algorithms like quicksort. If the data does not fit, we need to use external sorting that is able to spill to disk as needed.

2 External Merge Sort

Phase #1: Sorting – Sort small chunks of data that fit in main memory, and then write back to disk.

Phase #2: Merge – Combine sorted subfiles into a larger single file.

Two-way Merge Sort

1. Pass #0: Reads every $B$ pages of the table into memory. Sorts them, and writes them back into disk. Each sorted set of pages is called a run.
2. Pass #1,2,3...: Recursively merges pairs of runs into runs twice as long.

Number of Passes: $1 + \lceil \log_2 N \rceil$
Total I/O Cost: $2N \times (# \text{ of passes})$

General ($K$-way) Merge Sort

1. Pass #0: Use $B$ buffer pages, produce $N/B$ sorted runs of size $B$.
2. Pass #1,2,3...: Recursively merge $B - 1$ runs.

Number of Passes = $1 + \lceil \log_{B-1} \frac{N}{B} \rceil$
Total I/O Cost: $2N \times (# \text{ of passes})$

3 Aggregations

An aggregation operator in a query plan collapses the values of one or more tuples into a single scalar value. There are two approaches for implementing an aggregation: (1) sorting and (2) hashing.

Sorting

First sort the tuples on the GROUP BY key(s). Can use either an in-memory sorting algorithm if everything fits in the buffer pool (e.g., quicksort) or the external merge sort algorithm if the size of the data exceeds memory.

The DBMS then performs a sequential scan over the sorted data to compute the aggregation. The output of the operator will be sorted on the keys.
Hashing

Hashing can be computationally cheaper than sorting for computing aggregations. The DBMS populates an ephemeral hash table as it scans the table. For each record, check whether there is already an entry in the hash table and perform the appropriate modification.

If the size of the hash table is too large to fit in memory, then the DBMS has to spill it to disk:

- **Phase #1: Partition** – Use a hash function $h_1$ to split tuples into partitions on disk based on target hash key. This will put all tuples that match into the same partition. The DBMS spills partitions to disk via output buffers.
- **Phase #2: ReHash** – For each partition on disk, read its pages into memory and build an in-memory hash table based on a second hash function $h_2$ (where $h_1 \neq h_2$). Then go through each bucket of this hash table to bring together matching tuples to compute the aggregation. Note that this assumes that each partition fits in memory.

During the ReHash phase, the DBMS can store pairs of the form (GroupByKey→RunningValue) to compute the aggregation. The contents of RunningValue depends on the aggregation function. To insert a new tuple into the hash table:

- If we find a matching GroupByKey, just update the RunningValue appropriately.
- Else insert a new (GroupByKey→RunningValue) pair.