

Hash Tables



Lecture #06



Database Systems
15-445/15-645
Fall 2018

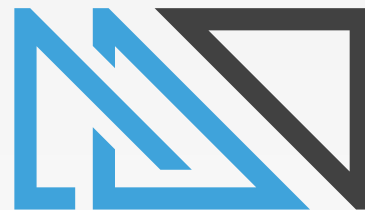
AP

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Computer Science
Carnegie Mellon Univ.

UPCOMING DATABASE EVENTS

MapD Talk

- Thursday Sept 20th @ 12:00pm
- CIC 4th Floor



MAPD

ADMINISTRIVIA

Project #1 is due Wednesday Sept 26th @ 11:59pm

Homework #2 is due Friday Sept 28th @ 11:59pm



REMINDER

If you have a question during the lecture, raise your hand and stop me.

Do **not** come up to the front after the lecture.

There are no stupid questions^(*).



COURSE STATUS

We are now going to talk about how to support the DBMS's execution engine to read/write data from pages.

Two types of data structures:

- Hash Tables
- Trees

Query Planning

Operator Execution

Access Methods

Buffer Pool Manager

Disk Manager

DATA STRUCTURES

Internal Meta-data

Core Data Storage

Temporary Data Structures

Table Indexes



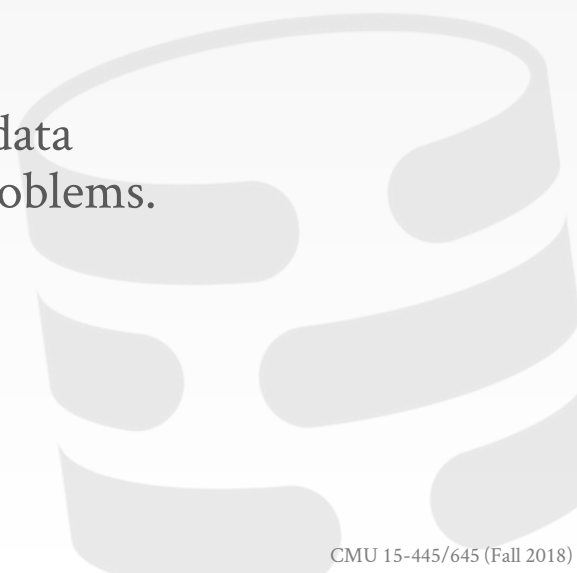
DESIGN DECISIONS

Data Organization

→ How we layout data structure in memory/pages and what information to store to support efficient access.

Concurrency

→ How to enable multiple threads to access the data structure at the same time without causing problems.



HASH TABLES

A hash table implements an associative array abstract data type that maps keys to values.

It uses a hash function to compute an offset into the array, from which the desired value can be found.



STATIC HASH TABLE

Allocate a giant array that has one slot for every element that you need to record.

To find an entry, mod the key by the number of elements to find the offset in the array.

hash(key)

0	abc
1	∅
2	def
	⋮
<i>n</i>	xyz

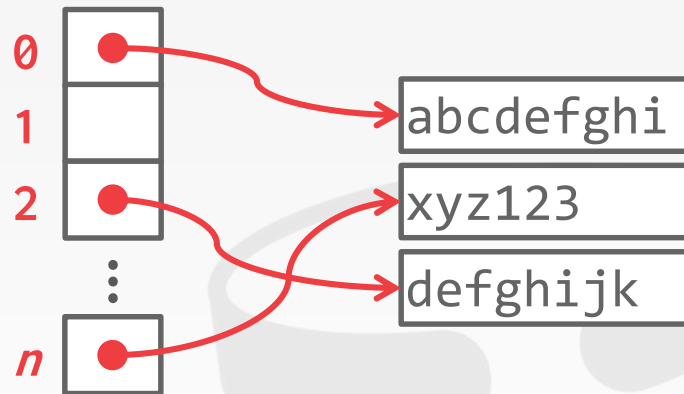


STATIC HASH TABLE

Allocate a giant array that has one slot for every element that you need to record.

To find an entry, mod the key by the number of elements to find the offset in the array.

hash(key)



ASSUMPTIONS

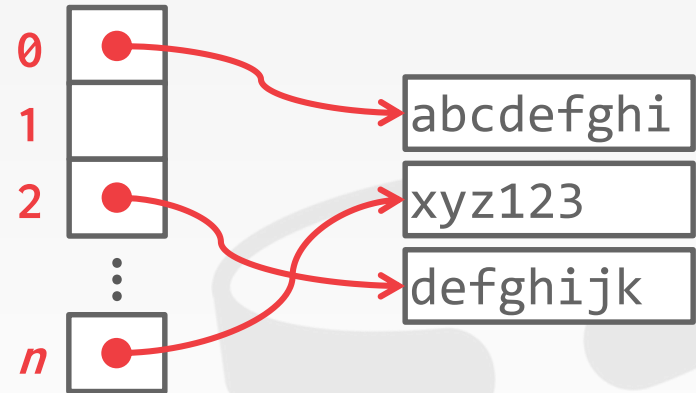
You know the number of elements ahead of time.

Each key is unique.

Perfect hash function.

→ If **key1** ≠ **key2**, then
hash(key1) ≠ hash(key2)

hash(key)



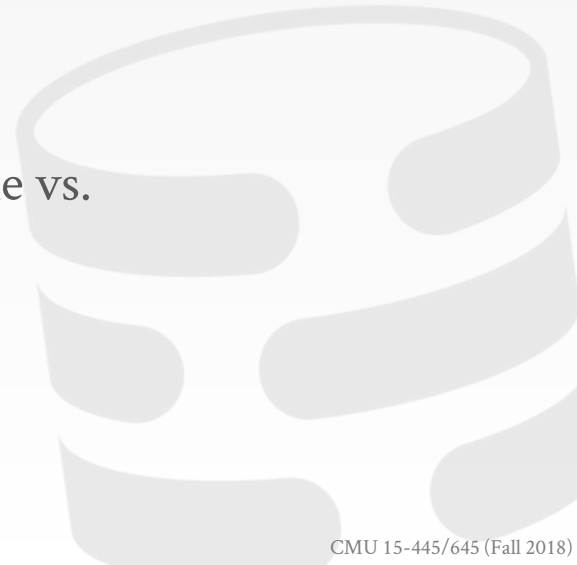
HASH TABLE

Design Decision #1: Hash Function

- How to map a large key space into a smaller domain.
- Trade-off between being fast vs. collision rate.

Design Decision #2: Hashing Scheme

- How to handle key collisions after hashing.
- Trade-off between allocating a large hash table vs. additional instructions to find/insert keys.



TODAY'S AGENDA

Hash Functions

Static Hashing Schemes

Dynamic Hashing Schemes



HASH FUNCTIONS

We don't want to use a cryptographic hash function for our join algorithm.

We want something that is fast and will have a low collision rate.



HASH FUNCTIONS

MurmurHash (2008)

→ Designed to a fast, general purpose hash function.

Google CityHash (2011)

→ Based on ideas from MurmurHash2

→ Designed to be faster for short keys (<64 bytes).

Google FarmHash (2014)

→ Newer version of CityHash with better collision rates.

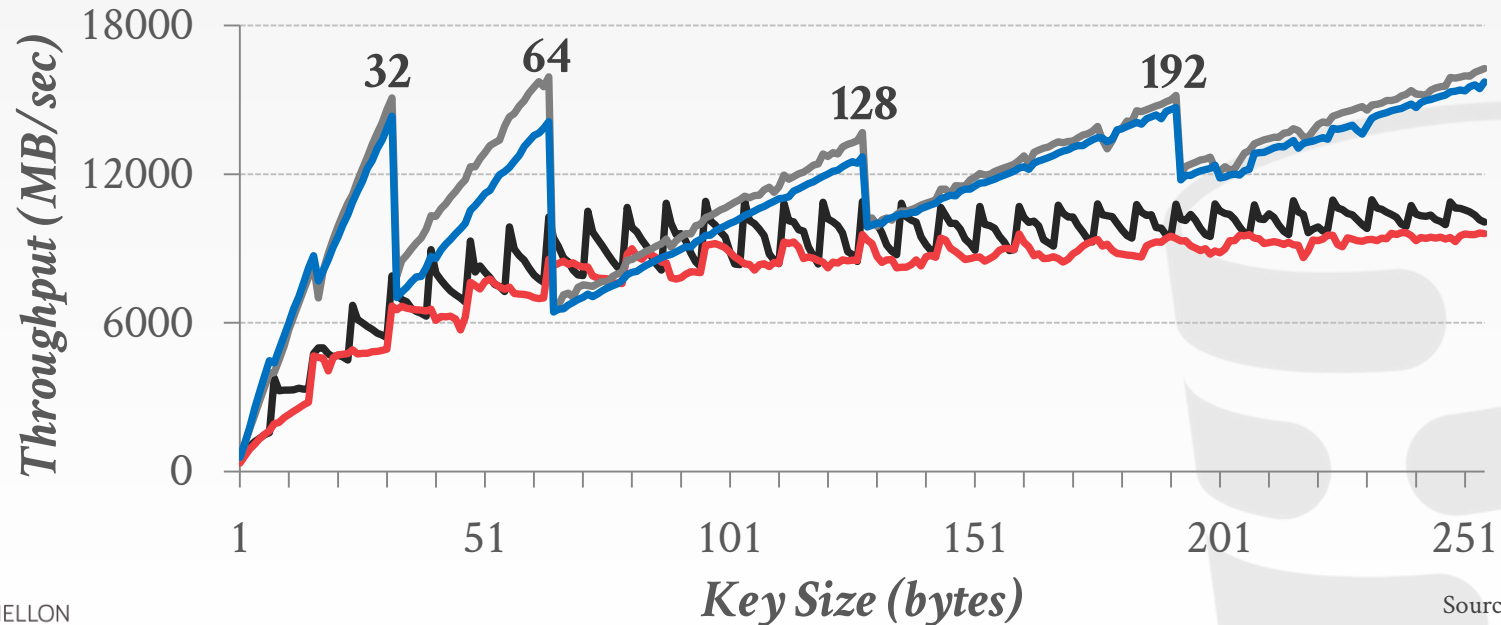
CLHash (2016)

→ Fast hashing function based on carry-less multiplication.

HASH FUNCTION BENCHMARKS

Intel Core i7-8700K @ 3.70GHz

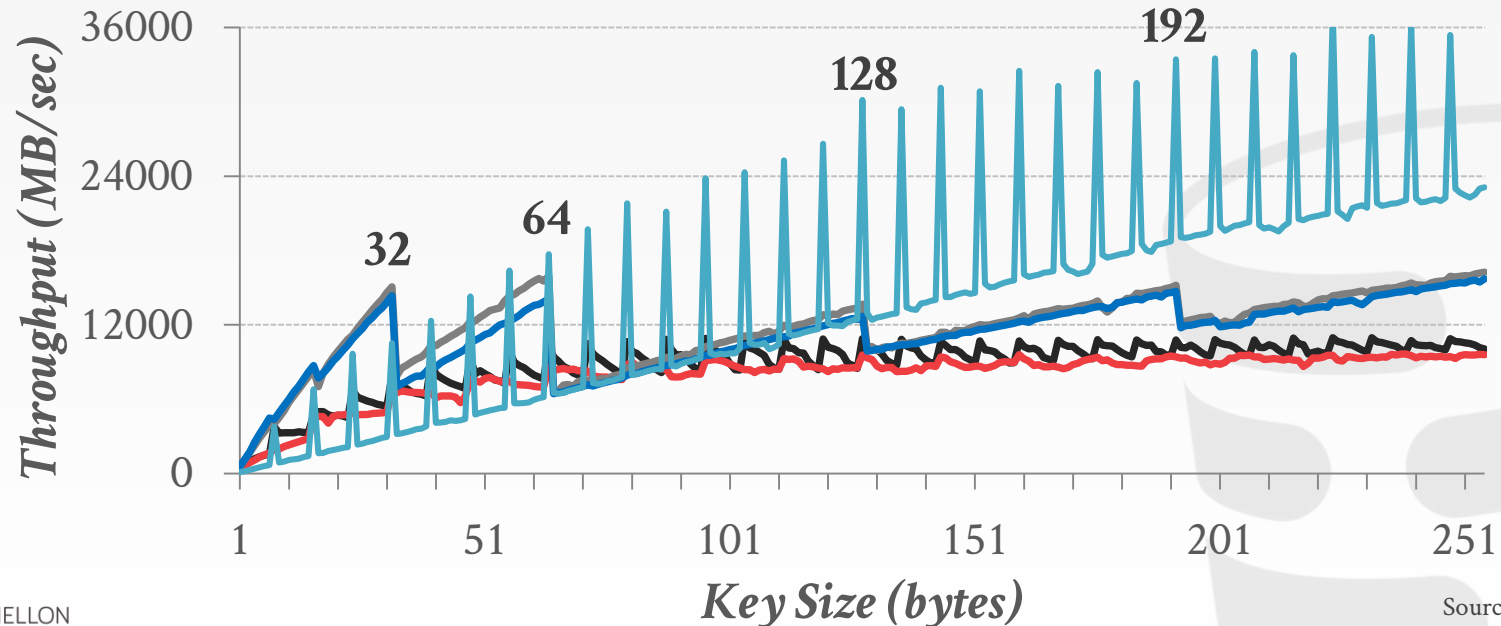
—std::hash —MurmurHash3 —CityHash —FarmHash —CLHash



HASH FUNCTION BENCHMARKS

Intel Core i7-8700K @ 3.70GHz

—std::hash —MurmurHash3 —CityHash —FarmHash —CLHash



STATIC HASHING SCHEMES

Approach #1: Linear Probe Hashing

Approach #2: Robin Hood Hashing

Approach #3: Cuckoo Hashing



LINEAR PROBE HASHING

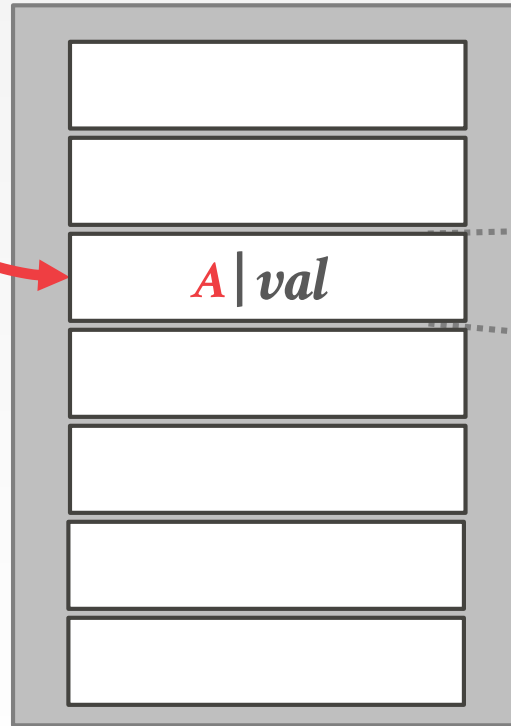
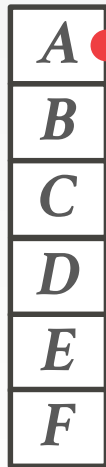
Single giant table of slots.

Resolve collisions by linearly searching for the next free slot in the table.

- To determine whether an element is present, hash to a location in the index and scan for it.
- Have to store the key in the index to know when to stop scanning.
- Insertions and deletions are generalizations of lookups.

LINEAR PROBE HASHING

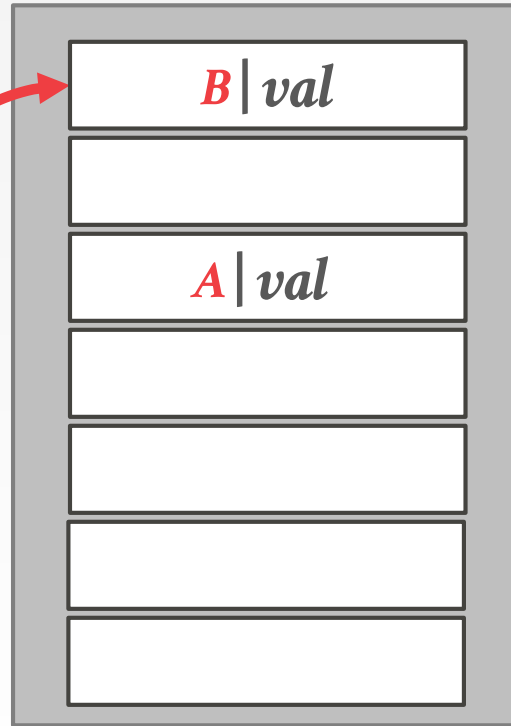
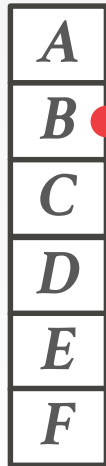
hash(key)



<key> | <value>

LINEAR PROBE HASHING

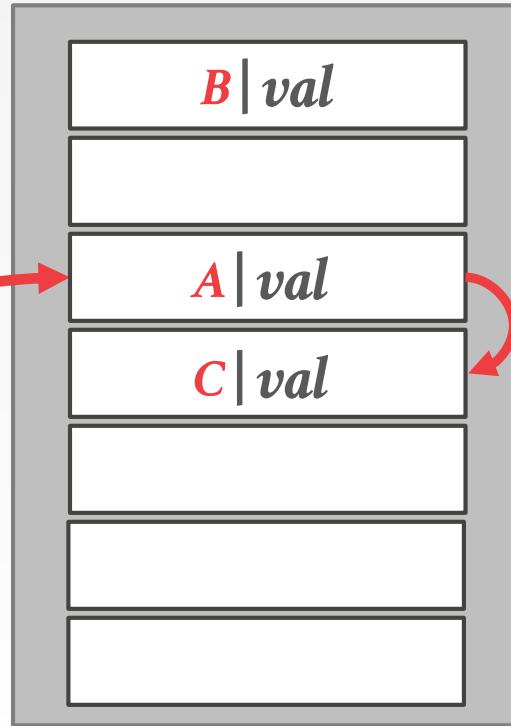
hash(key)



LINEAR PROBE HASHING

hash(key)

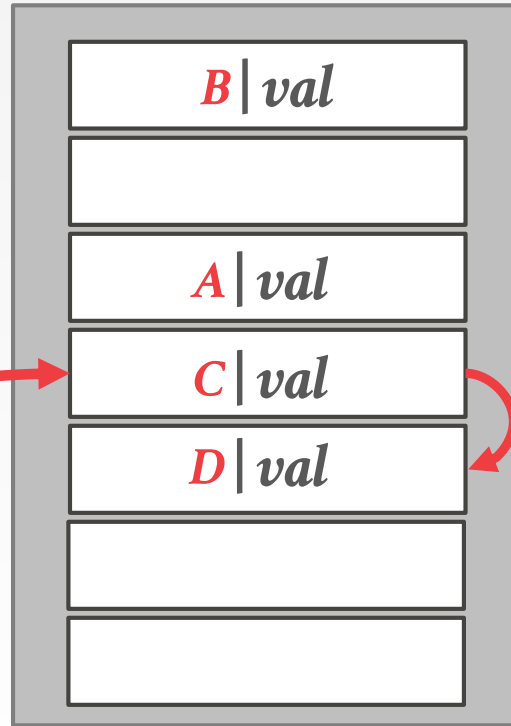
A
B
C
D
E
F



LINEAR PROBE HASHING

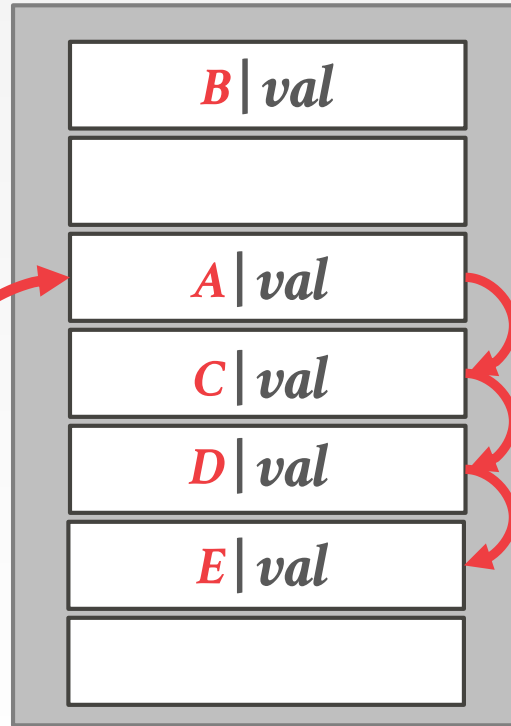
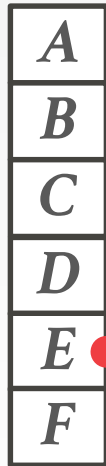
hash(key)

A
B
C
D
E
F



LINEAR PROBE HASHING

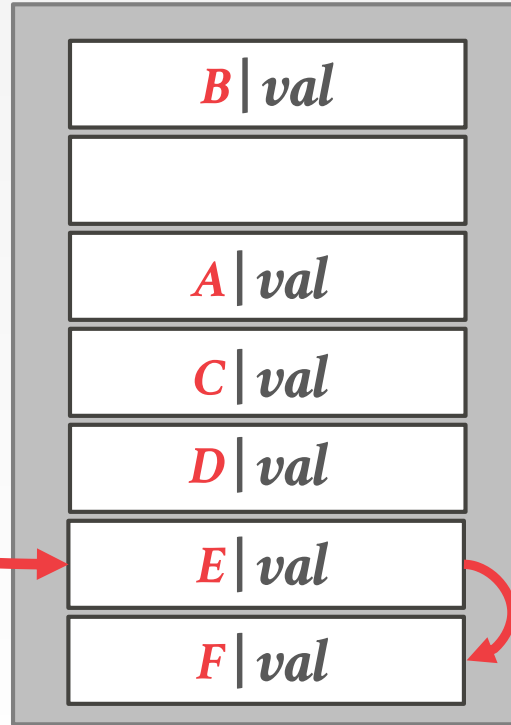
hash(key)



LINEAR PROBE HASHING

hash(key)

A
B
C
D
E
F



NON-UNIQUE KEYS

Choice #1: Separate Linked List

→ Store values in separate storage area for each key.



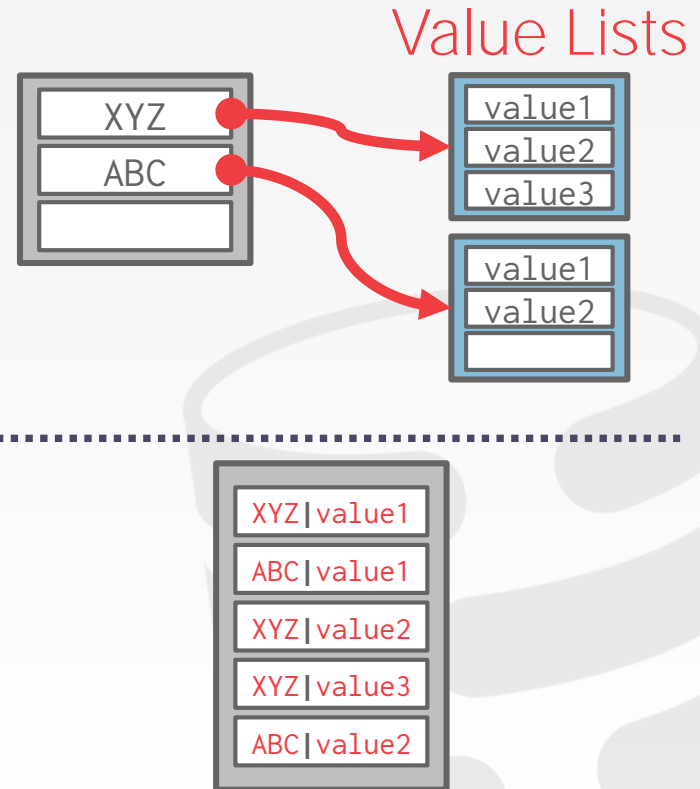
NON-UNIQUE KEYS

Choice #1: Separate Linked List

→ Store values in separate storage area for each key.

Choice #2: Redundant Keys

→ Store duplicate keys entries together in the hash table.



OBSERVATION

To reduce the # of wasteful comparisons, it is important to avoid collisions of hashed keys.

This requires a hash table with $\sim 2x$ the number of slots as the number of elements.



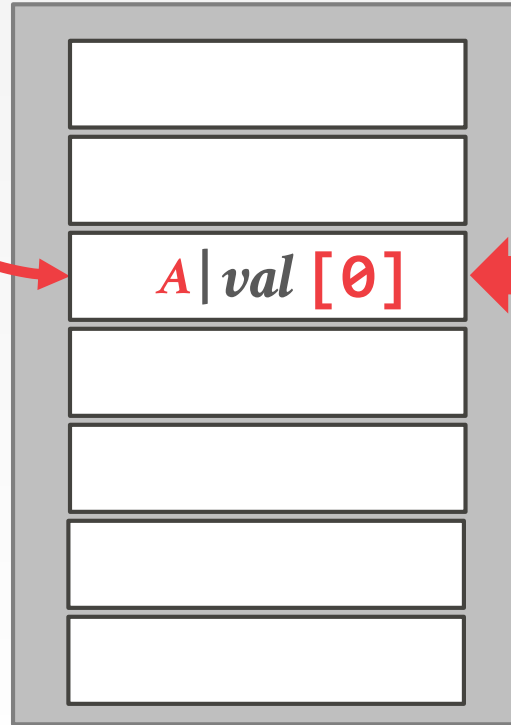
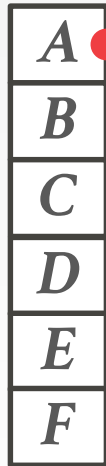
ROBIN HOOD HASHING

Variant of linear probe hashing that steals slots from "rich" keys and give them to "poor" keys.

- Each key tracks the number of positions they are from where its optimal position in the table.
- On insert, a key takes the slot of another key if the first key is farther away from its optimal position than the second key.

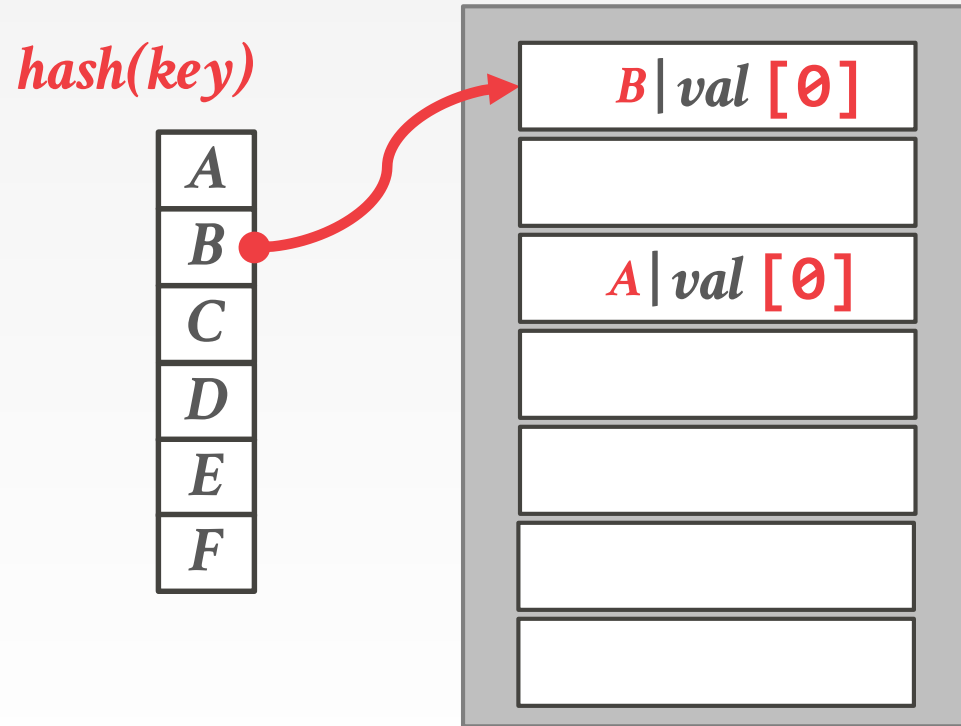
ROBIN HOOD HASHING

hash(key)



of "Jumps" From First Position

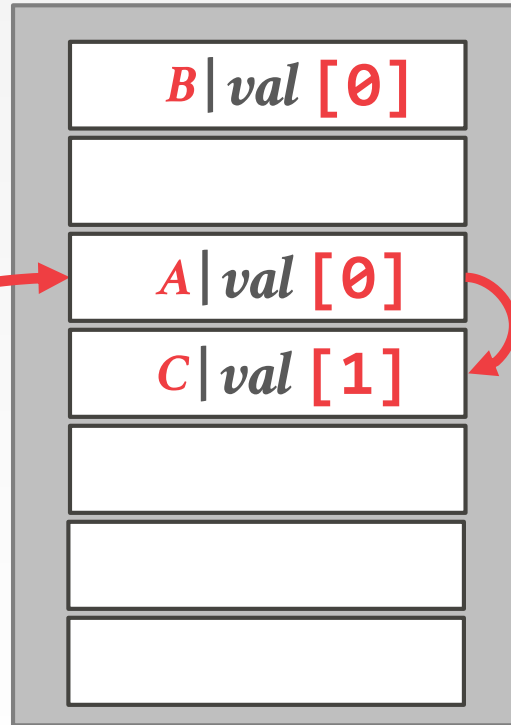
ROBIN HOOD HASHING



ROBIN HOOD HASHING

hash(key)

A
B
C
D
E
F

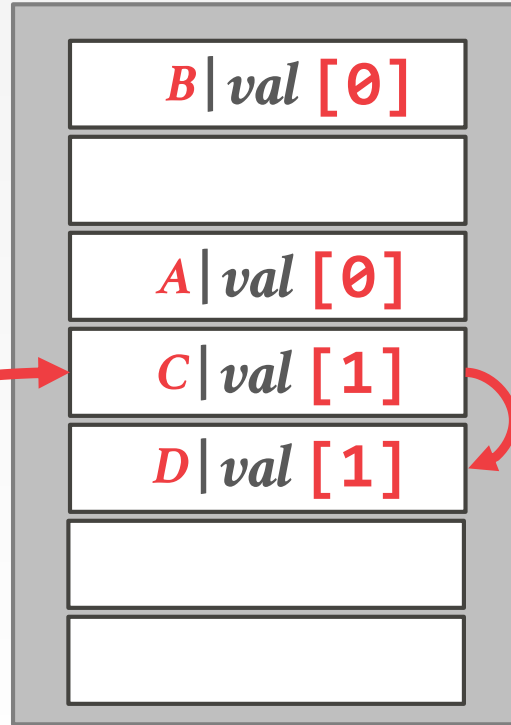


$A[0] == C[0]$

ROBIN HOOD HASHING

hash(key)

A
B
C
D
E
F

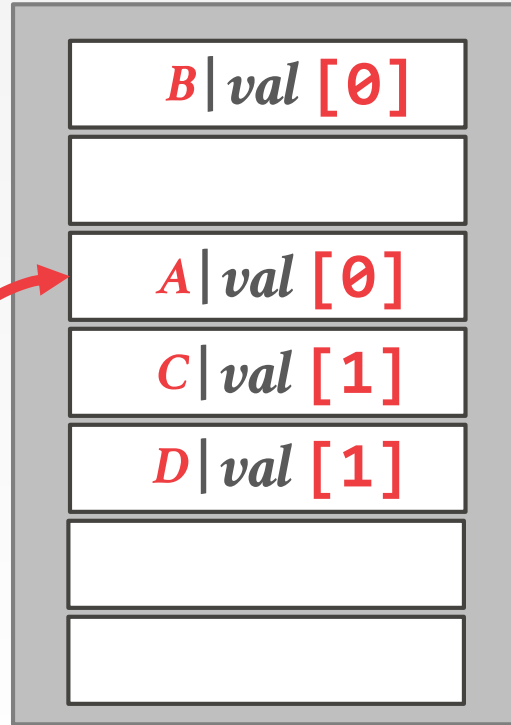


C[*1*] > *D*[*0*]

ROBIN HOOD HASHING

hash(key)

A
B
C
D
E
F



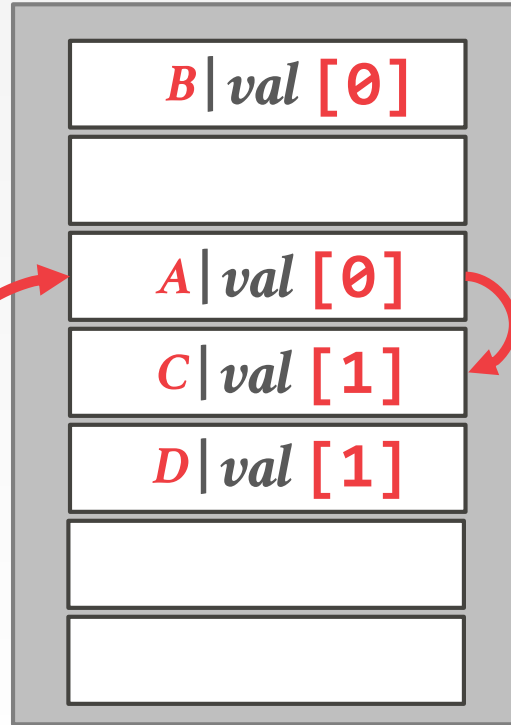
A[*0*] == *E*[*0*]



ROBIN HOOD HASHING

hash(key)

A
B
C
D
E
F



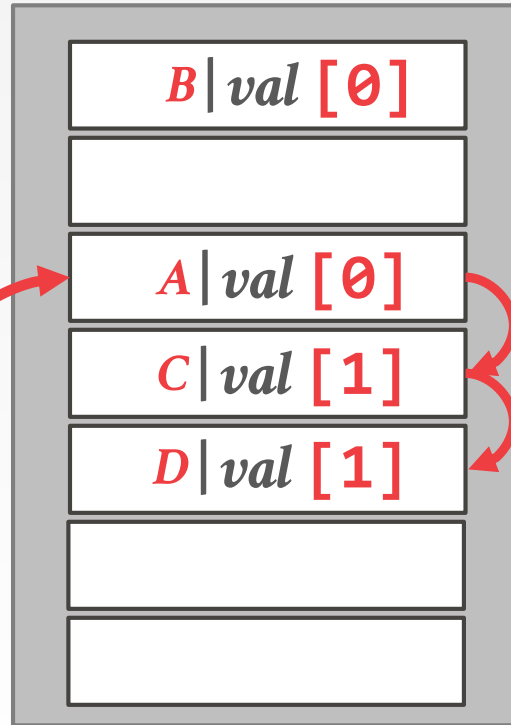
$A[0] == E[0]$

$C[1] == E[1]$

ROBIN HOOD HASHING

hash(key)

A
B
C
D
E
F



$A[0] == E[0]$

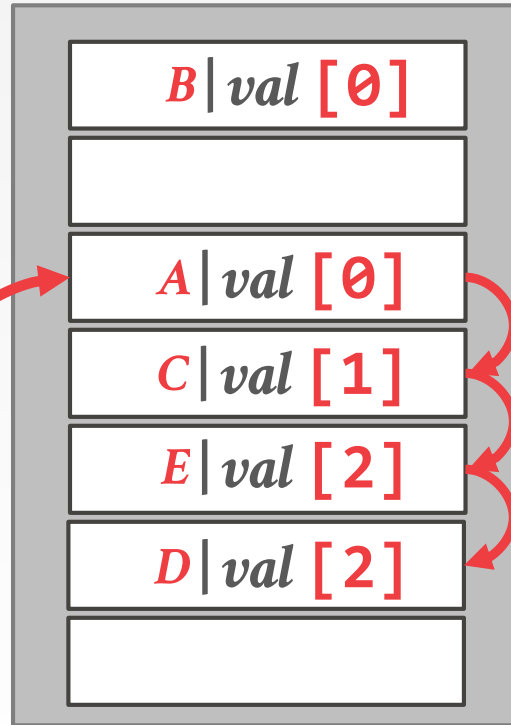
$C[1] == E[1]$

$D[1] < E[2]$

ROBIN HOOD HASHING

hash(key)

A
B
C
D
E
F



$A[0] == E[0]$

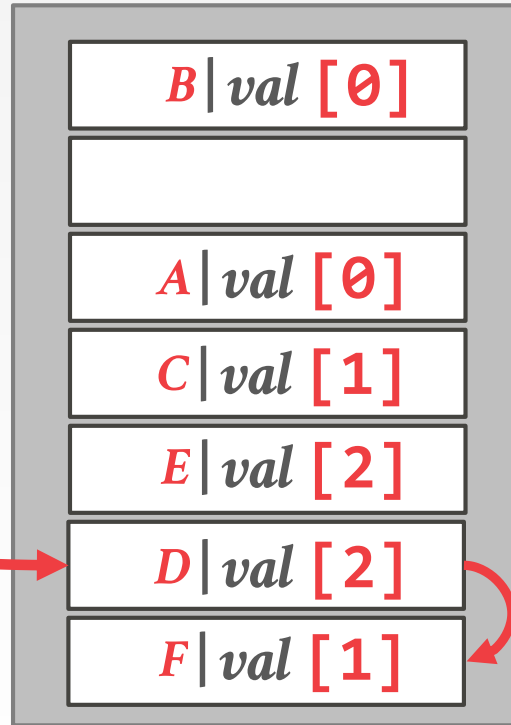
$C[1] == E[1]$

$D[1] < E[2]$

ROBIN HOOD HASHING

hash(key)

A
B
C
D
E
F



$D[2] > F[0]$

CUCKOO HASHING

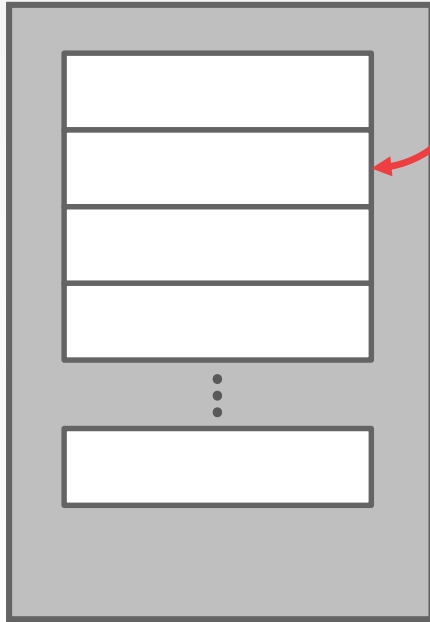
Use multiple hash tables with different hash functions.

- On insert, check every table and pick anyone that has a free slot.
- If no table has a free slot, evict the element from one of them and then re-hash it find a new location.

Look-ups and deletions are always $O(1)$ because only one location per hash table is checked.

CUCKOO HASHING

Hash Table #1

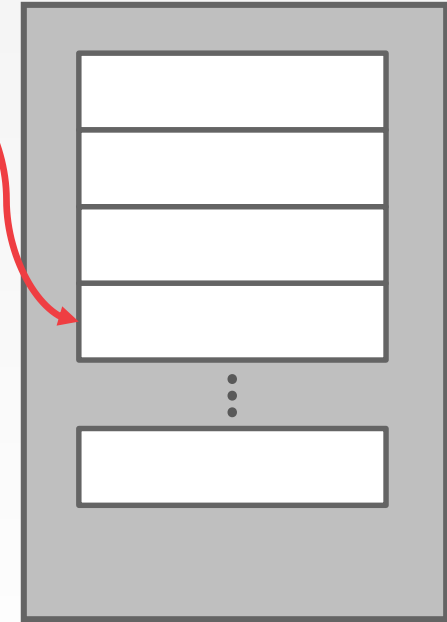


Insert A

$hash_1(A)$

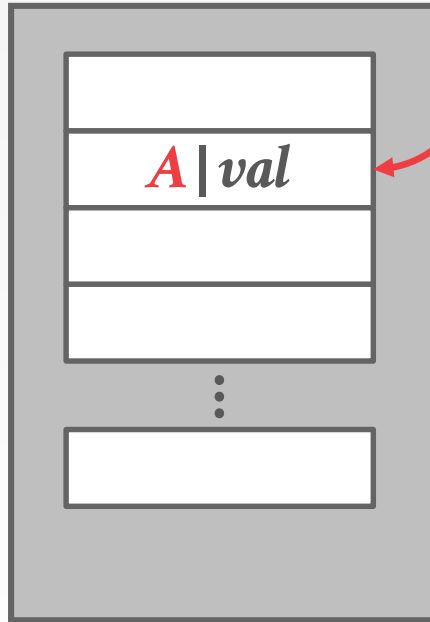
$hash_2(A)$

Hash Table #2



CUCKOO HASHING

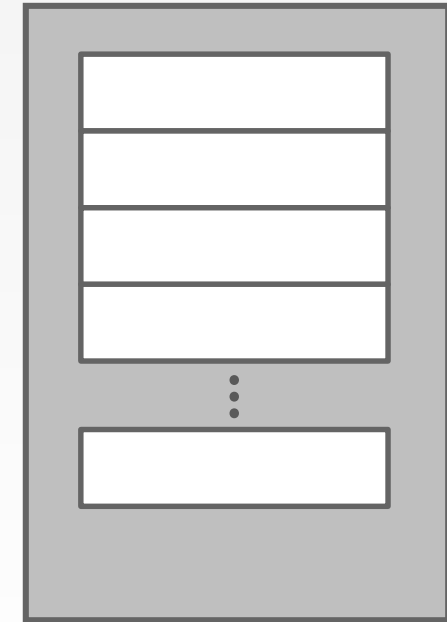
Hash Table #1



Insert A

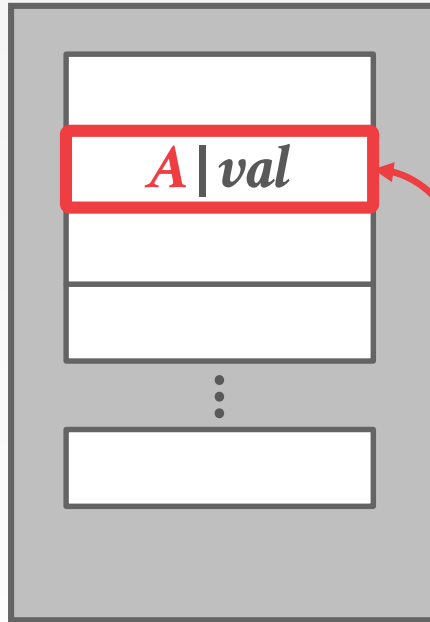
$\text{hash}_1(A)$ $\text{hash}_2(A)$

Hash Table #2



CUCKOO HASHING

Hash Table #1



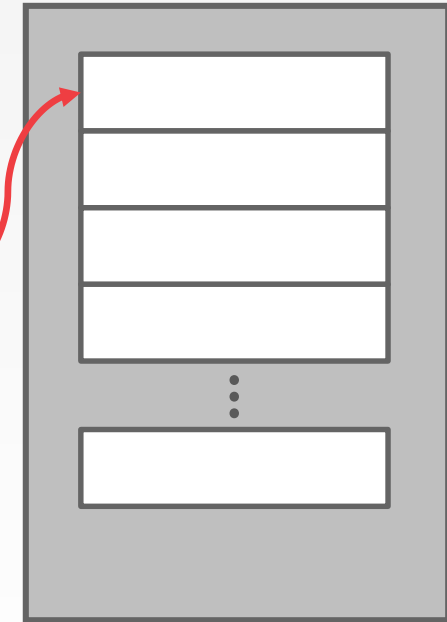
Insert A

$hash_1(A)$ $hash_2(A)$

Insert B

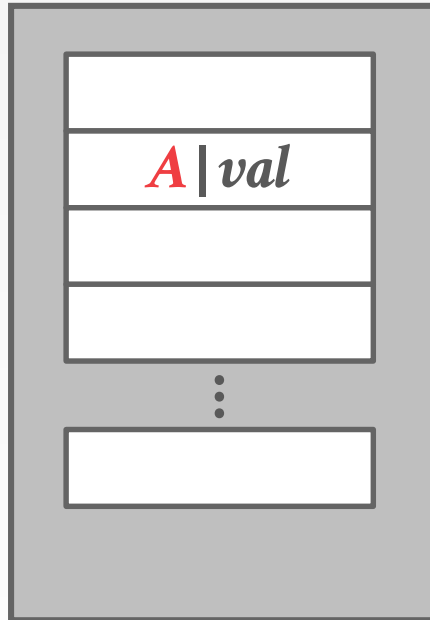
$hash_1(B)$ $hash_2(B)$

Hash Table #2



CUCKOO HASHING

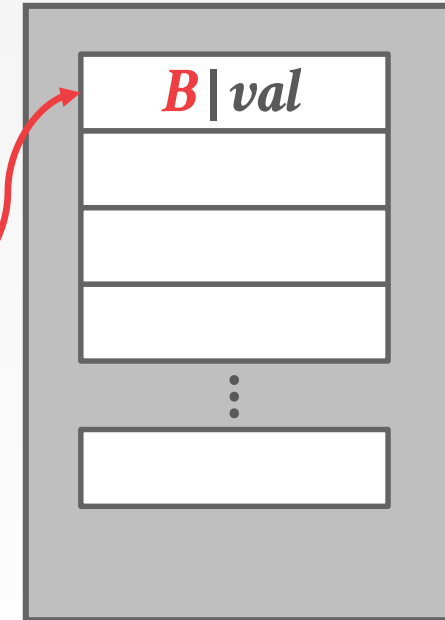
Hash Table #1



Insert A
 $\text{hash}_1(A)$ $\text{hash}_2(A)$

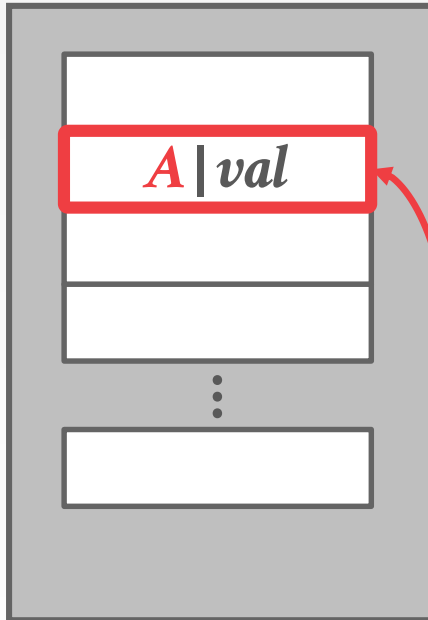
Insert B
 $\text{hash}_1(B)$ $\text{hash}_2(B)$

Hash Table #2



CUCKOO HASHING

Hash Table #1



Insert A

$\text{hash}_1(A)$ $\text{hash}_2(A)$

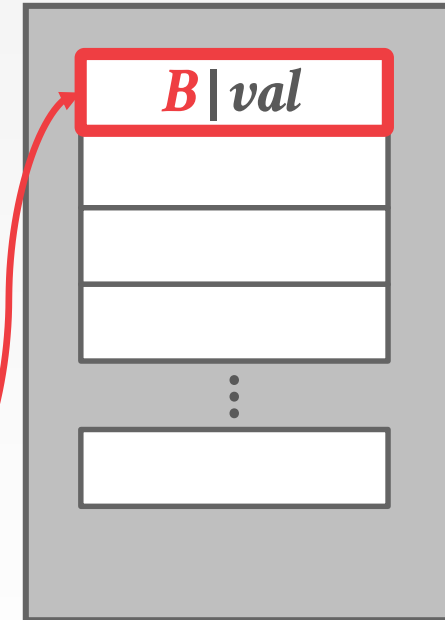
Insert B

$\text{hash}_1(B)$ $\text{hash}_2(B)$

Insert C

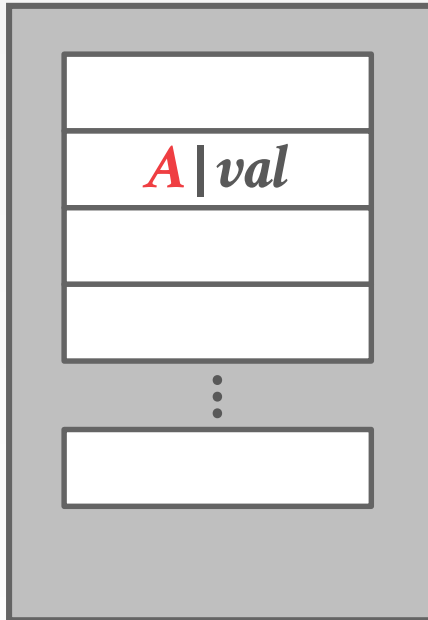
$\text{hash}_1(C)$ $\text{hash}_2(C)$

Hash Table #2



CUCKOO HASHING

Hash Table #1



Insert A

$\text{hash}_1(A)$ $\text{hash}_2(A)$

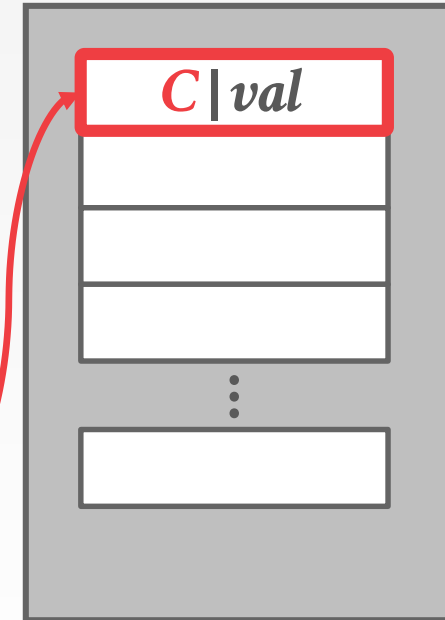
Insert B

$\text{hash}_1(B)$ $\text{hash}_2(B)$

Insert C

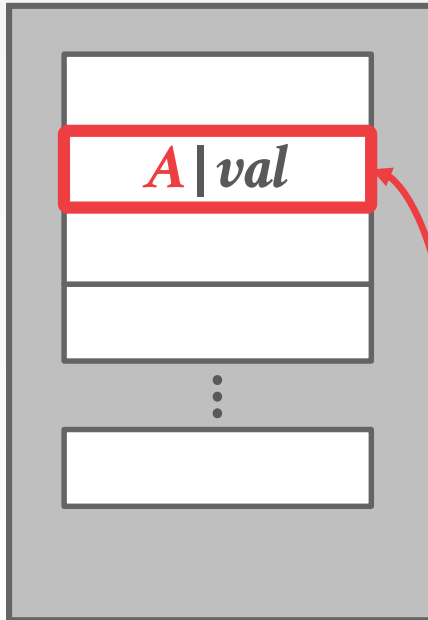
$\text{hash}_1(C)$ $\text{hash}_2(C)$

Hash Table #2



CUCKOO HASHING

Hash Table #1



Insert A

$\text{hash}_1(A)$ $\text{hash}_2(A)$

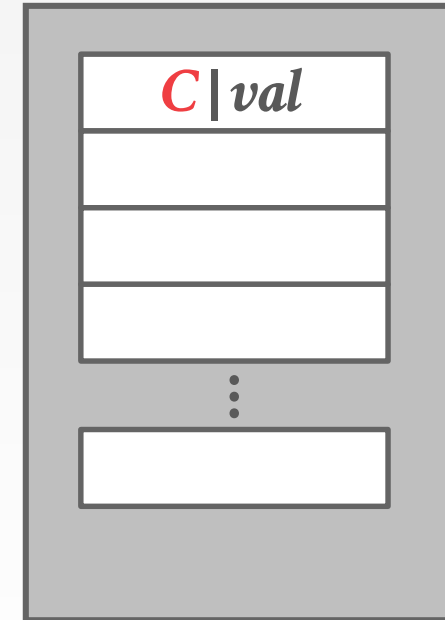
Insert B

$\text{hash}_1(B)$ $\text{hash}_2(B)$

Insert C

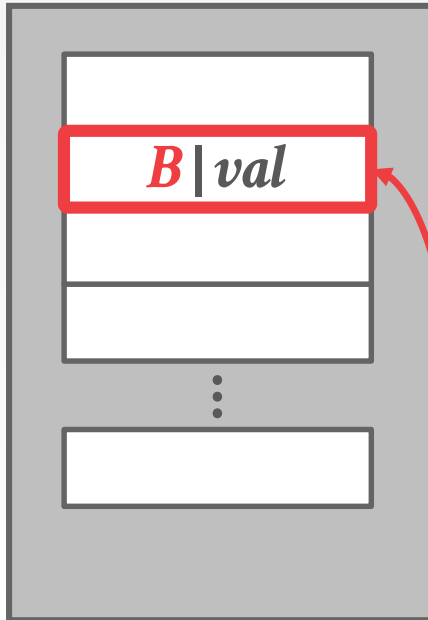
$\text{hash}_1(C)$ $\text{hash}_2(C)$
 $\text{hash}_1(B)$

Hash Table #2



CUCKOO HASHING

Hash Table #1



Insert A

$hash_1(A)$ $hash_2(A)$

Insert B

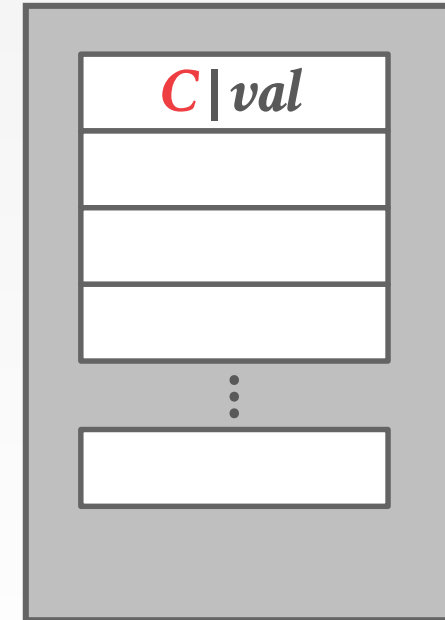
$hash_1(B)$ $hash_2(B)$

Insert C

$hash_1(C)$ $hash_2(C)$

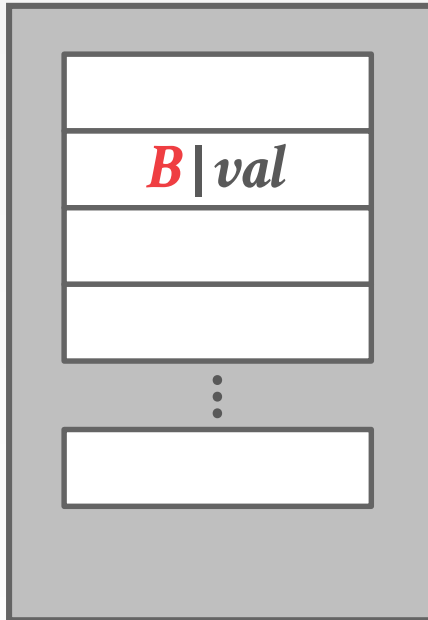
$hash_1(B)$

Hash Table #2



CUCKOO HASHING

Hash Table #1



Insert A

$\text{hash}_1(A)$ $\text{hash}_2(A)$

Insert B

$\text{hash}_1(B)$ $\text{hash}_2(B)$

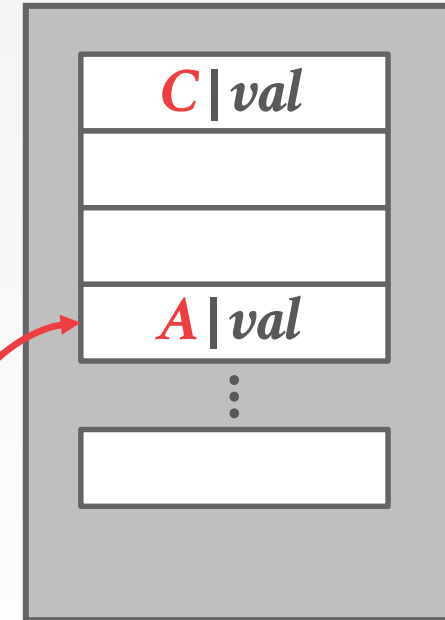
Insert C

$\text{hash}_1(C)$ $\text{hash}_2(C)$

$\text{hash}_1(B)$

$\text{hash}_2(A)$

Hash Table #2



CUCKOO HASHING

Make sure that we don't get stuck in an infinite loop when moving keys.

If we find a cycle, then we can rebuild the entire hash tables with new hash functions.

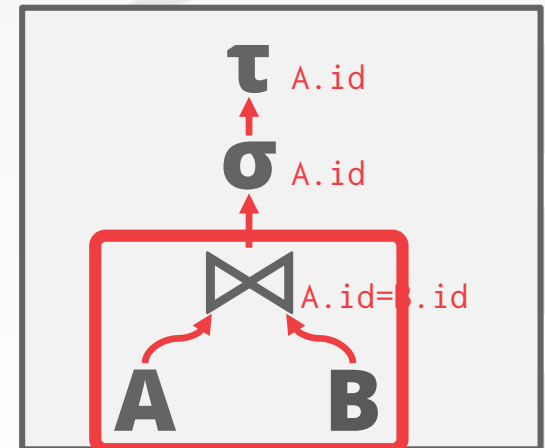
- With **two** hash functions, we (probably) won't need to rebuild the table until it is at about 50% full.
- With **three** hash functions, we (probably) won't need to rebuild the table until it is at about 90% full.

OBSERVATION

The previous hash tables require knowing the number of elements you want to store ahead of time.

→ Otherwise you have rebuild the entire table if you need to grow/shrink.

```
SELECT A.id
FROM A, B
WHERE A.id = B.id
ORDER BY A.id
```



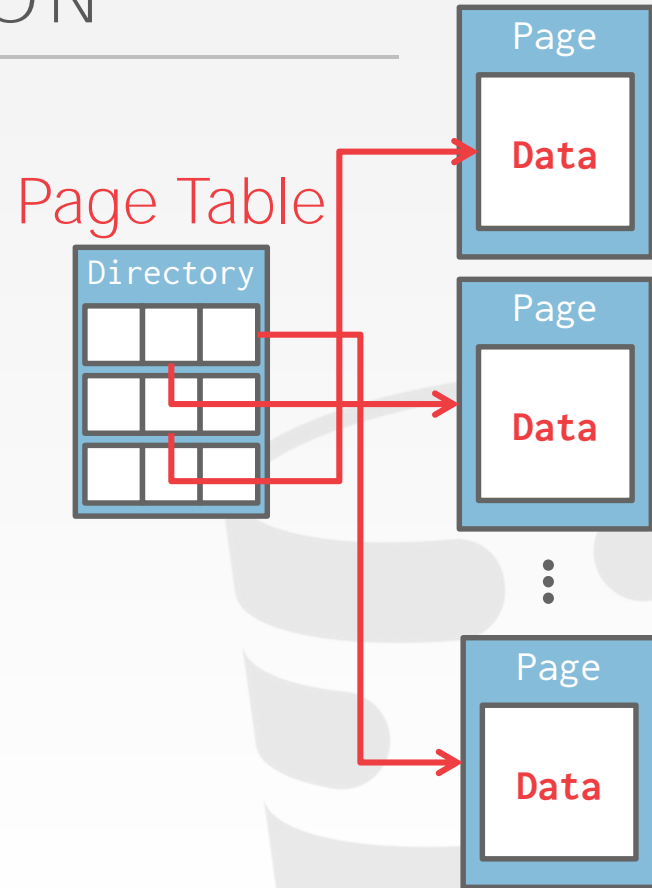
OBSERVATION

The previous hash tables require knowing the number of elements you want to store ahead of time.

→ Otherwise you have rebuild the entire table if you need to grow/shrink.

Dynamic hash tables are able to grow/shrink on demand.

→ Extensible Hashing
→ Linear Hashing

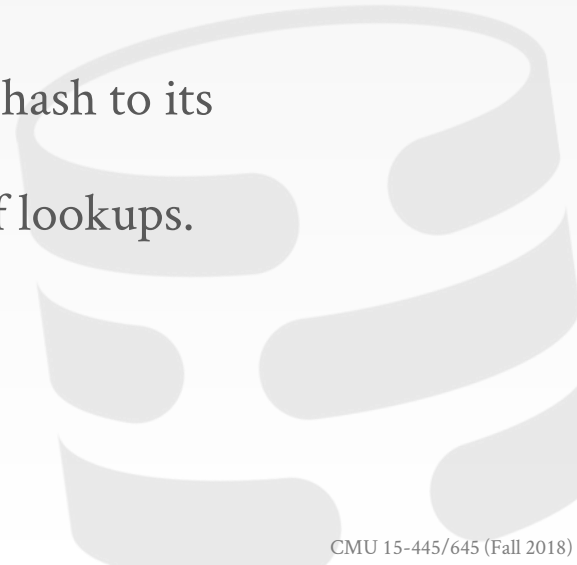


CHAINED HASHING

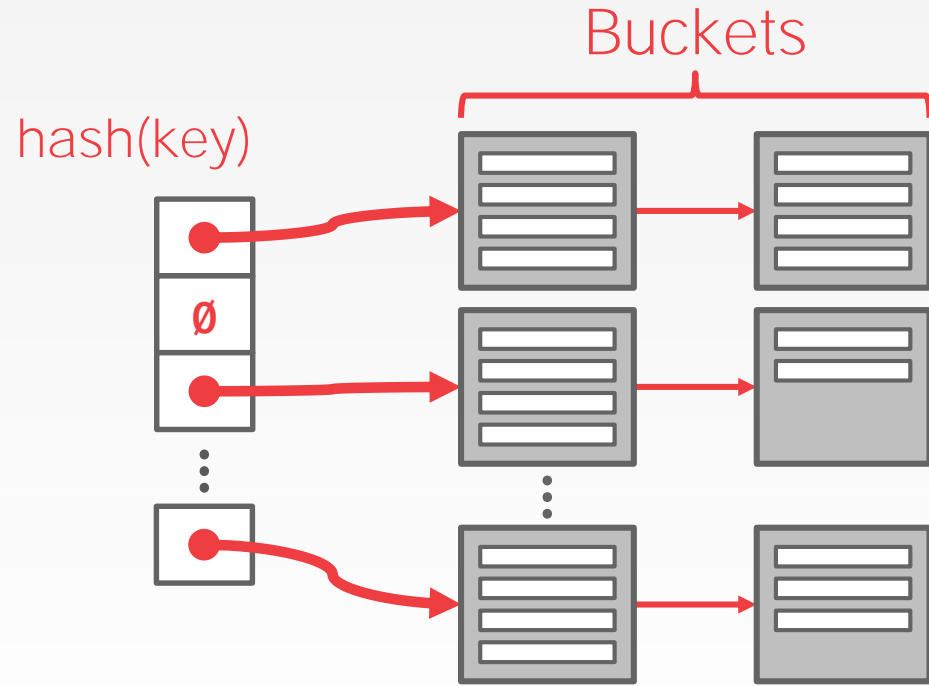
Maintain a linked list of buckets for each slot in the hash table.

Resolve collisions by placing all elements with the same hash key into the same bucket.

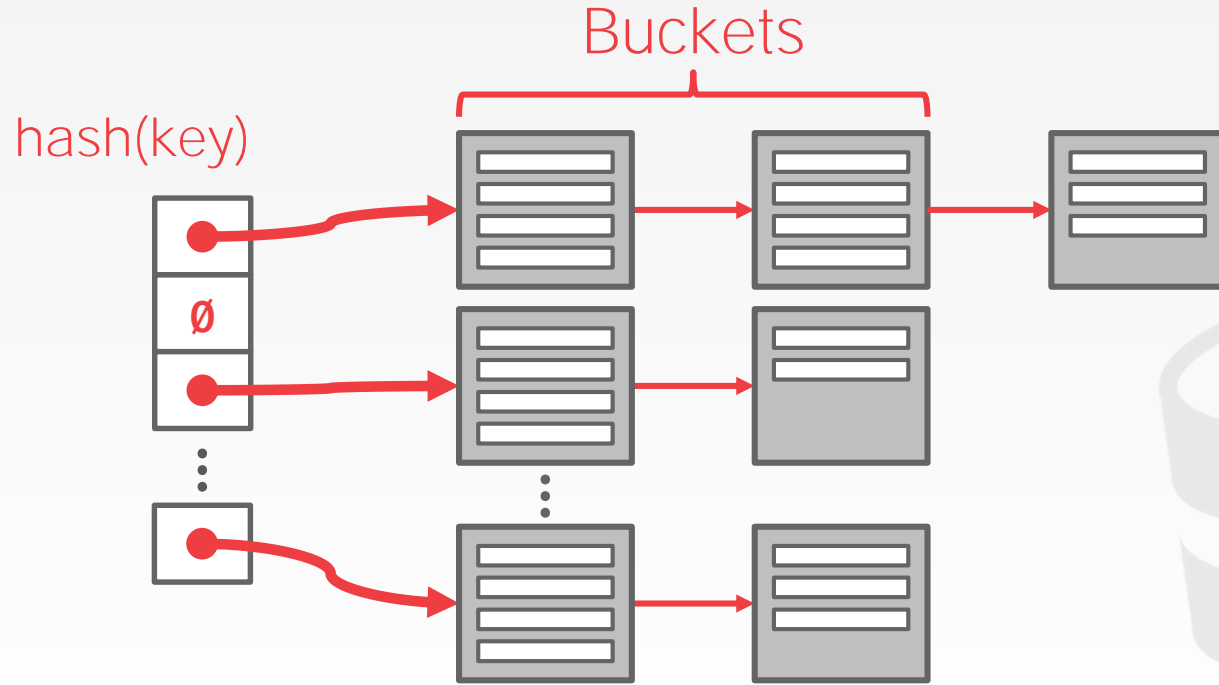
- To determine whether an element is present, hash to its bucket and scan for it.
- Insertions and deletions are generalizations of lookups.



CHAINED HASHING



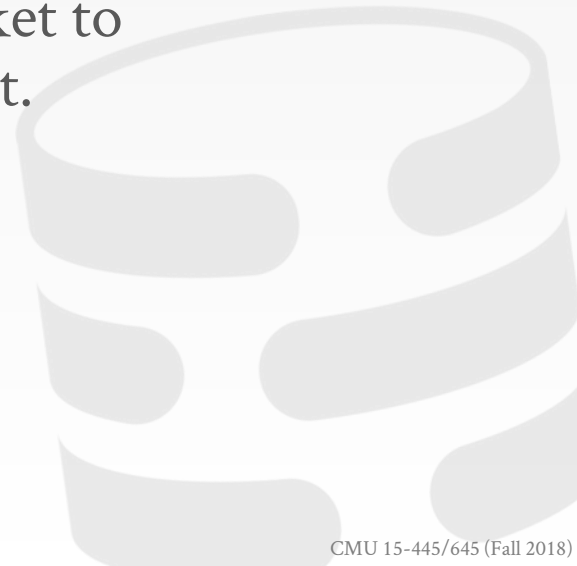
CHAINED HASHING



CHAINED HASHING

The hash table can grow infinitely because you just keep adding new buckets to the linked list.

You only need to take a latch on the bucket to store a new entry or extend the linked list.



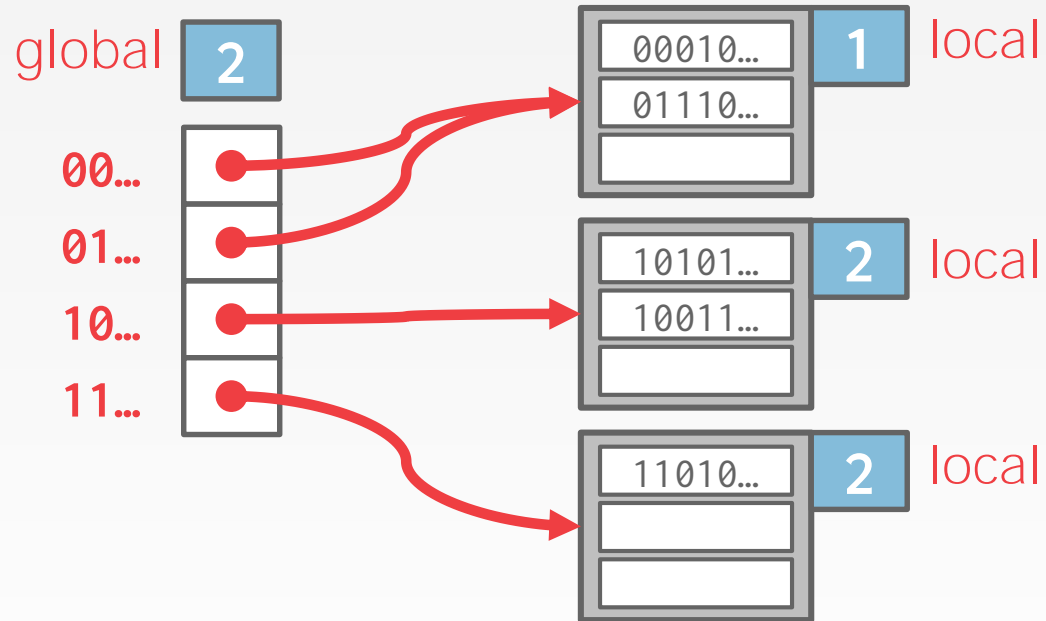
EXTENDIBLE HASHING

Chained-hashing approach where we split buckets instead of letting the linked list grow forever.

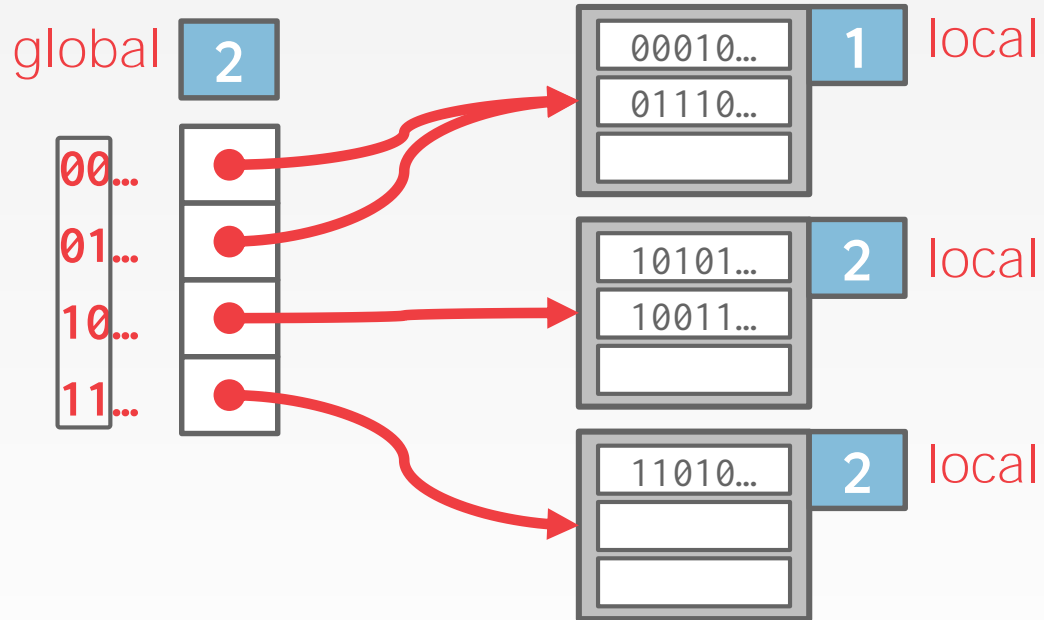
This requires reshuffling entries on split, but the change is localized.



EXTENDIBLE HASHING



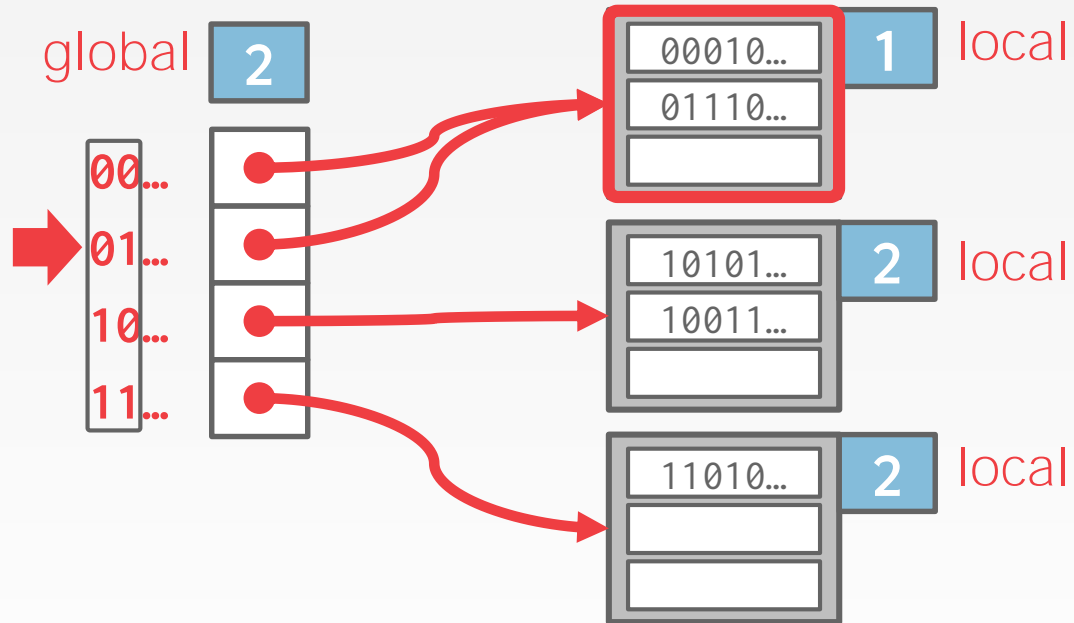
EXTENDIBLE HASHING



Find A
hash(A) = **01**110...



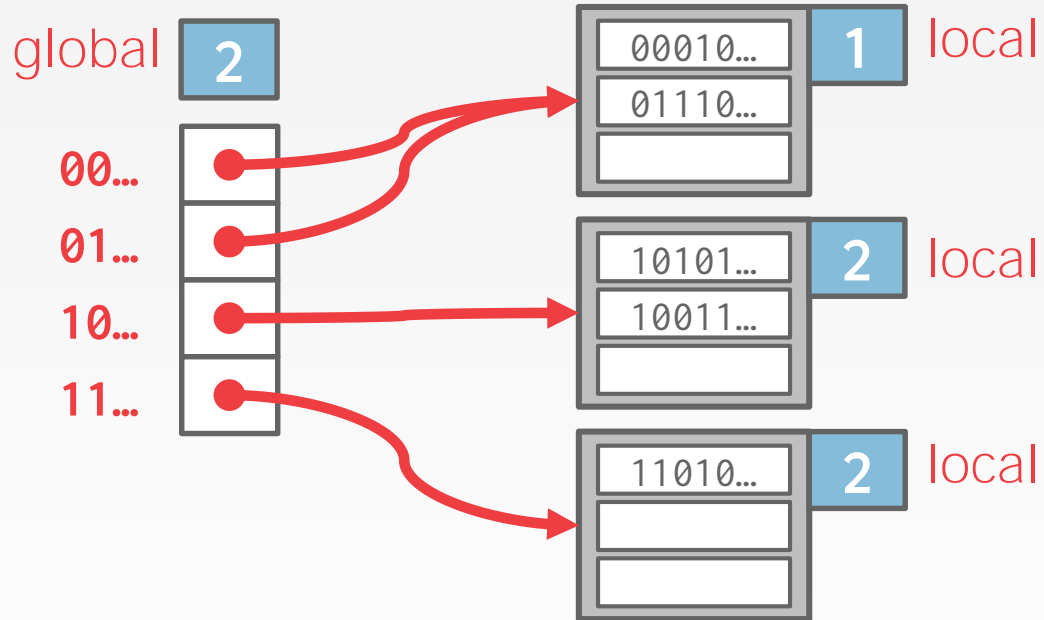
EXTENDIBLE HASHING



Find A
 $\text{hash}(A) = 01110\dots$



EXTENDIBLE HASHING



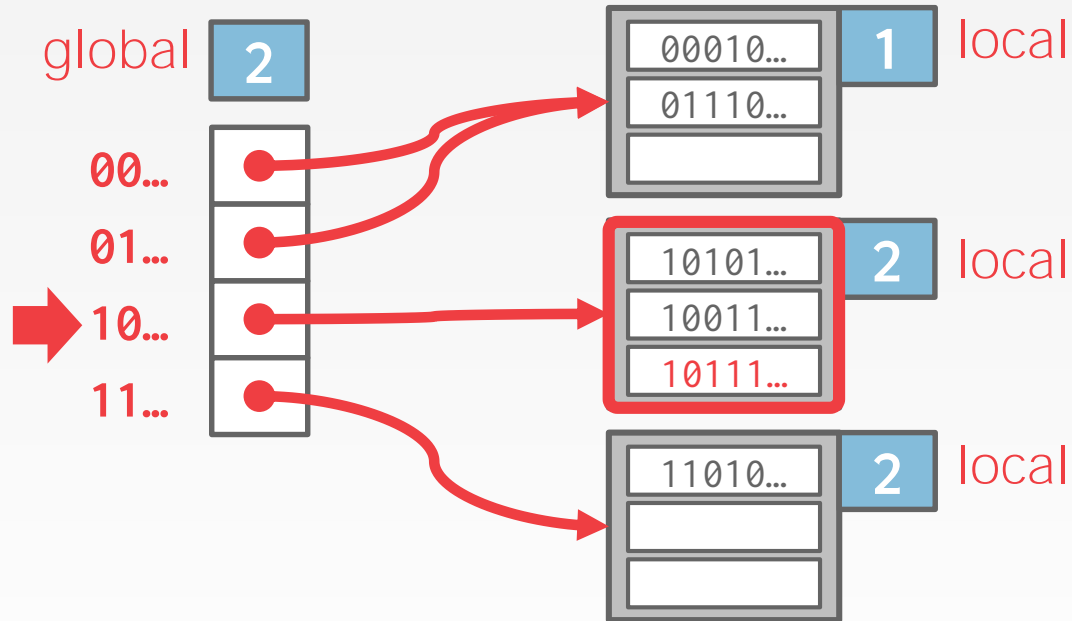
Find A

hash(A) = **01110...**

Insert B

hash(B) = **10111...**

EXTENDIBLE HASHING



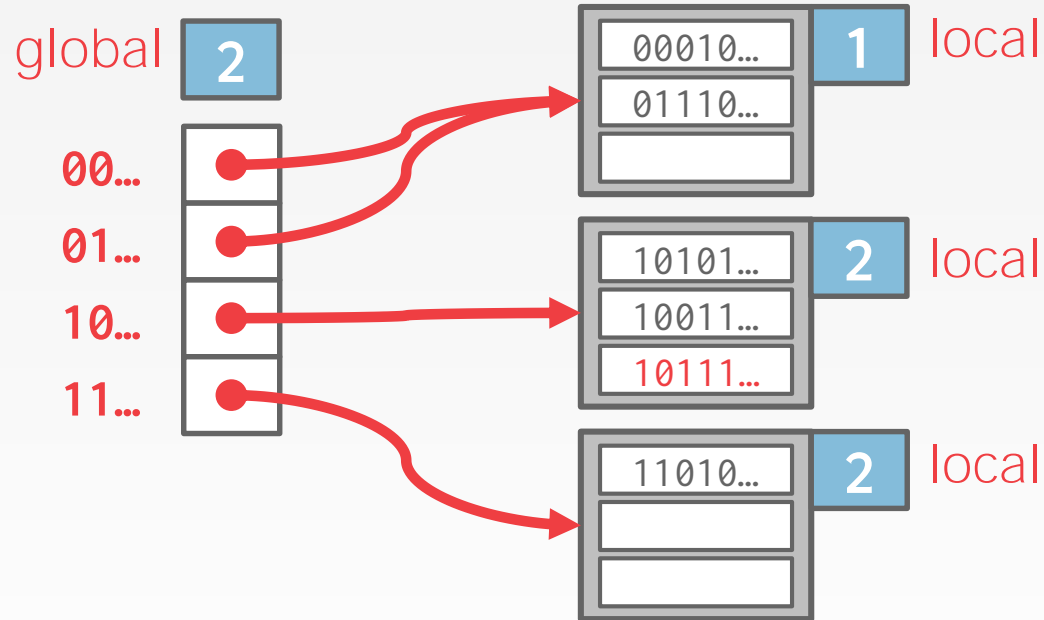
Find A

hash(A) = **01110...**

Insert B

hash(B) = **10111...**

EXTENDIBLE HASHING



Find A

hash(A) = **01110...**

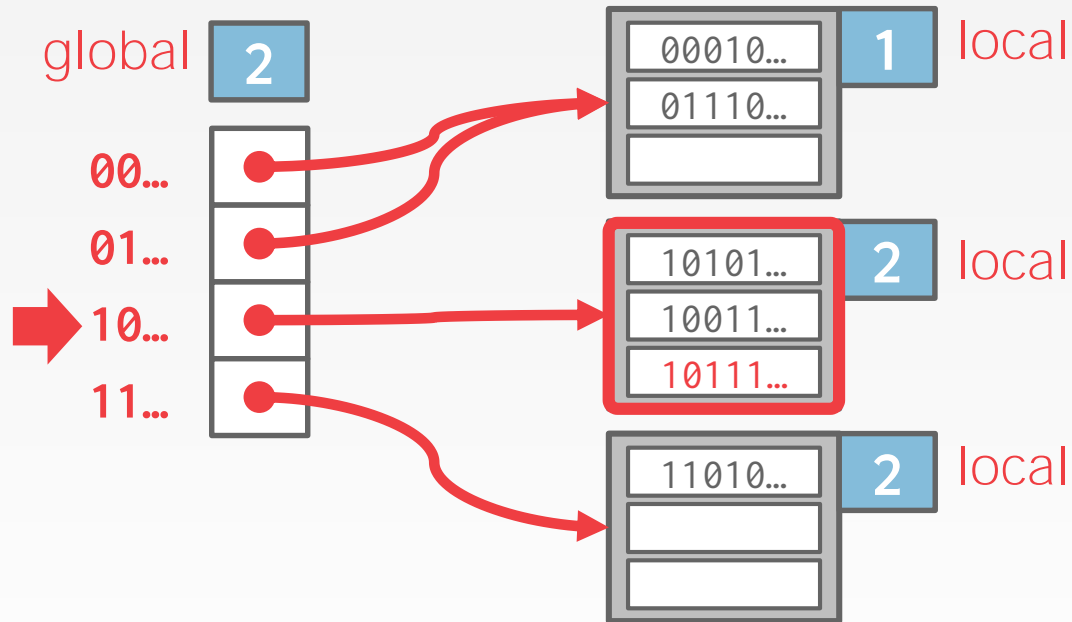
Insert B

hash(B) = **10111...**

Insert C

hash(C) = **10100...**

EXTENDIBLE HASHING



Find A

hash(A) = **01110...**

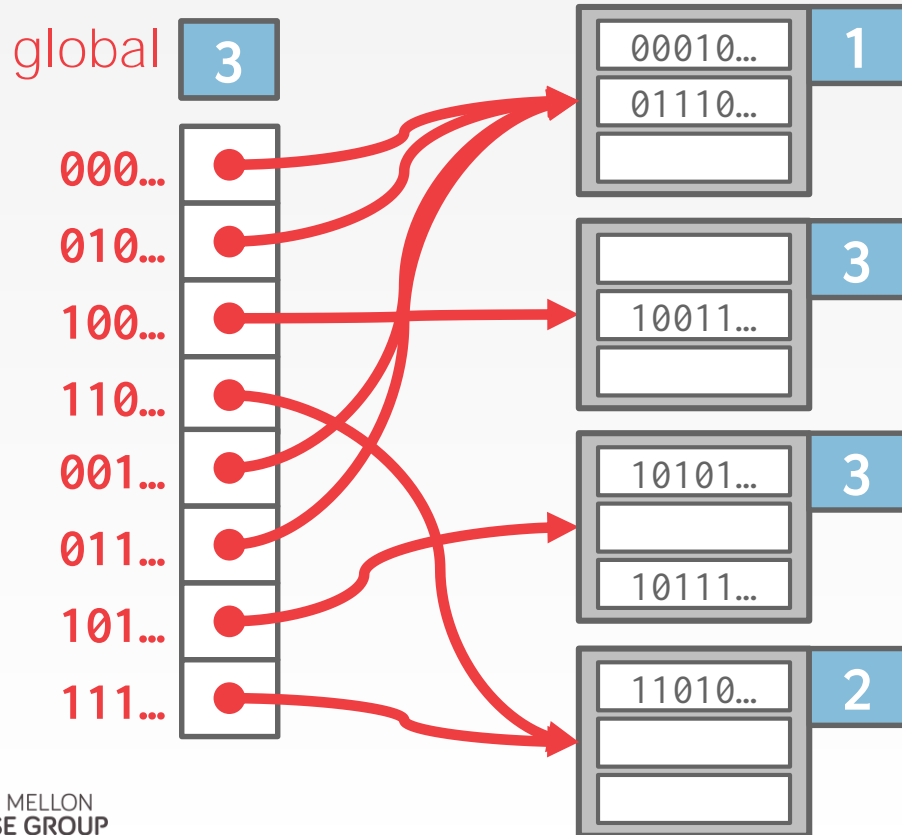
Insert B

hash(B) = **10111...**

Insert C

hash(C) = **10100...**

EXTENDIBLE HASHING



Find A

hash(A) = **01110...**

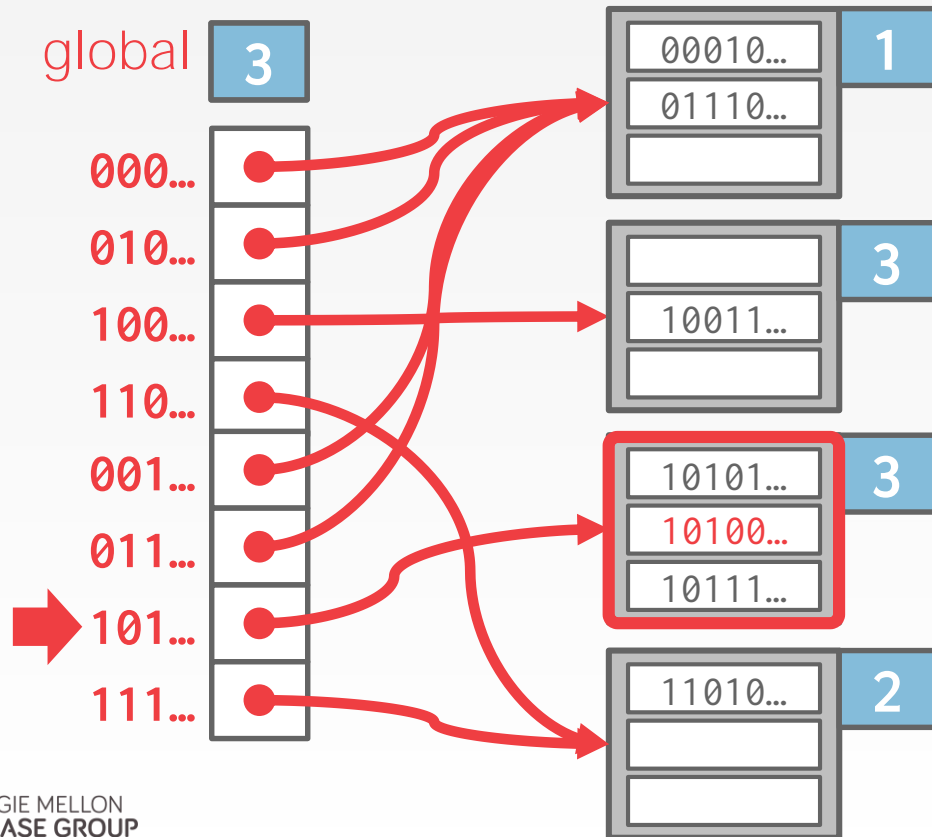
Insert B

hash(B) = **10111...**

Insert C

hash(C) = **10100...**

EXTENDIBLE HASHING



Find A

hash(A) = **01110...**

Insert B

hash(B) = **10111...**

Insert C

hash(C) = **10100...**

LINEAR HASHING

Maintain a pointer that tracks the next bucket to split.

When any bucket overflows, split the bucket at the pointer location.

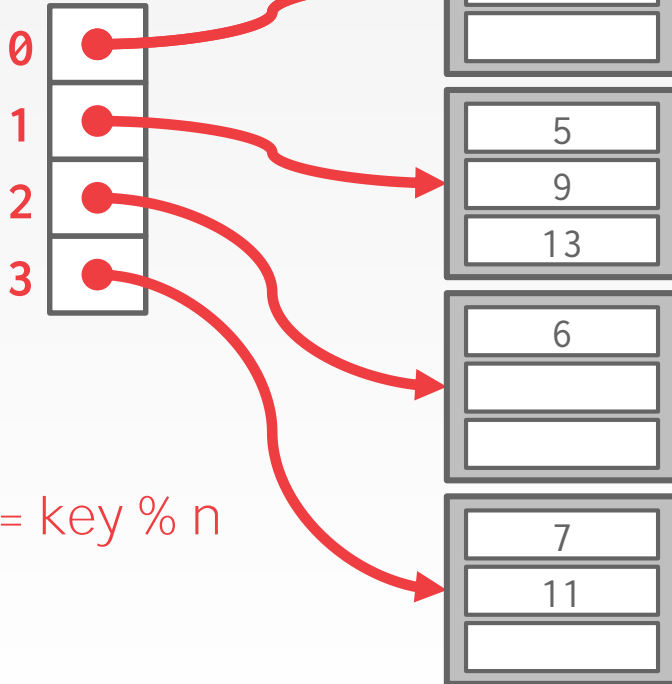
Overflow criterion is left up to the implementation.

- Space Utilization
- Average Length of Overflow Chains



LINEAR HASHING

Split
Pointer

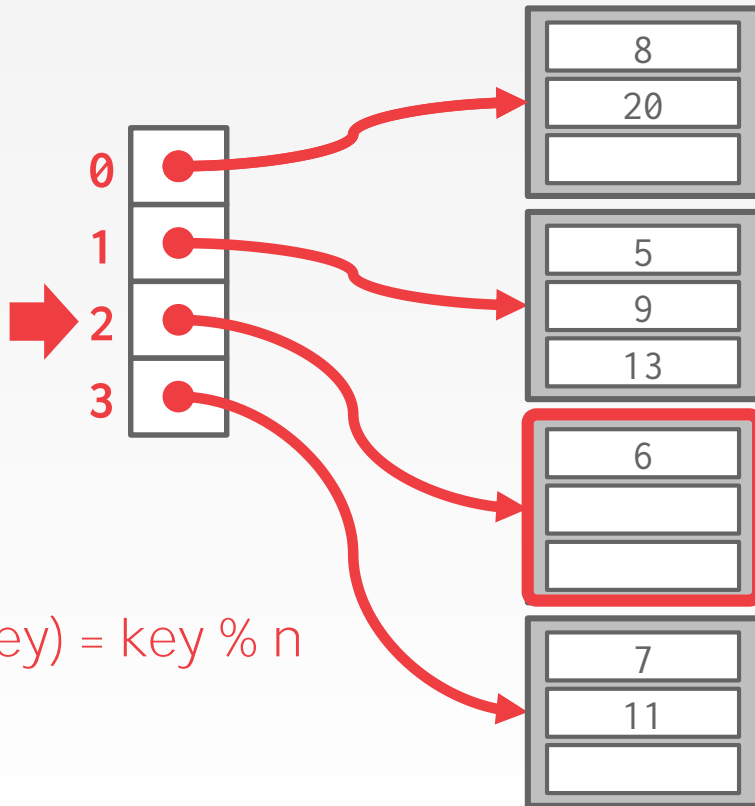


$$\text{hash}_1(\text{key}) = \text{key} \% n$$



LINEAR HASHING

Split
Pointer



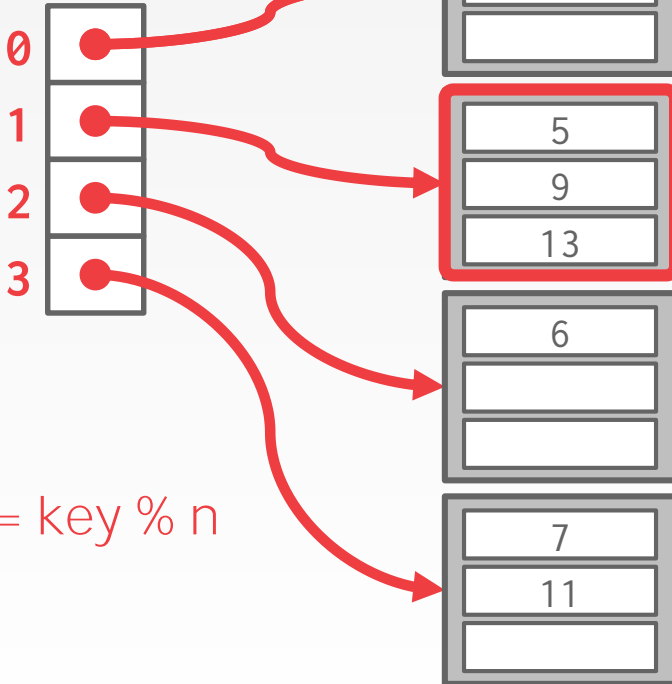
$$\text{hash}_1(\text{key}) = \text{key} \% n$$

Find 6
 $\text{hash}_1(6) = 6 \% 4 = 2$



LINEAR HASHING

Split
Pointer



$$\text{hash}_1(\text{key}) = \text{key} \% n$$

Find 6

$$\text{hash}_1(6) = 6 \% 4 = 2$$

Insert 17

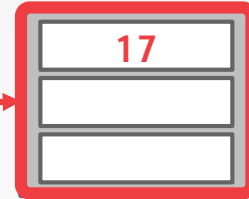
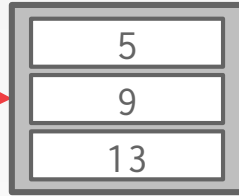
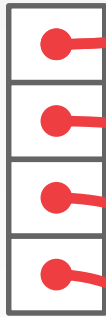
$$\text{hash}_1(17) = 17 \% 4 = 1$$

LINEAR HASHING

Split
Pointer



0
1
2
3



Overflow!

Find 6

$$\text{hash}_1(6) = 6\%4 = 2$$

Insert 17

$$\text{hash}_1(17) = 17\%4 = 1$$

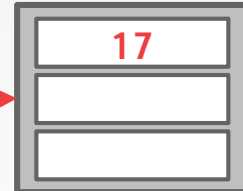
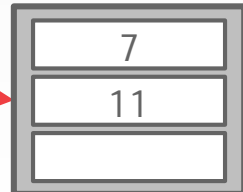
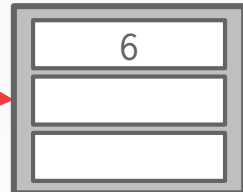
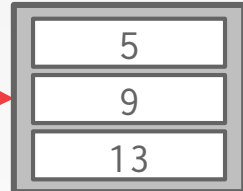
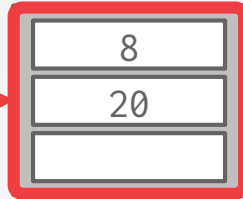
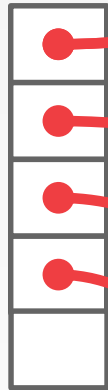
$$\text{hash}_1(\text{key}) = \text{key} \% n$$

LINEAR HASHING

Split
Pointer



0
1
2
3
4



Overflow!

Find 6

$$\text{hash}_1(6) = 6\%4 = 2$$

Insert 17

$$\text{hash}_1(17) = 17\%4 = 1$$

$$\text{hash}_1(\text{key}) = \text{key} \% n$$

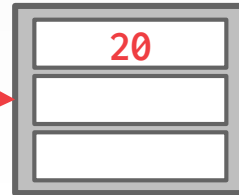
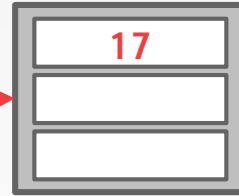
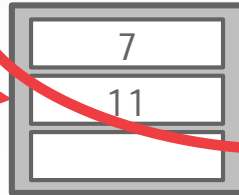
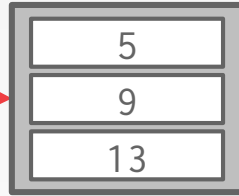
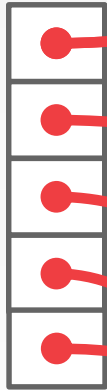
$$\text{hash}_2(\text{key}) = \text{key} \% 2n$$

LINEAR HASHING

Split
Pointer



0
1
2
3
4



Find 6

$$\text{hash}_1(6) = 6\%4 = 2$$

Insert 17

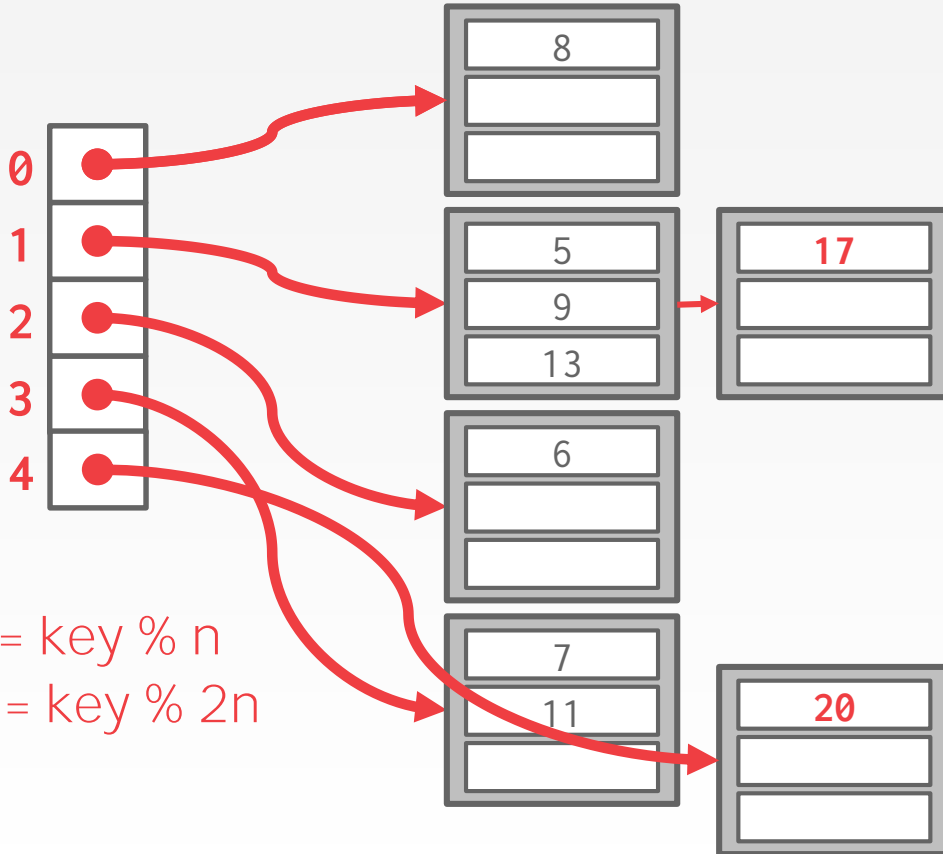
$$\text{hash}_1(17) = 17\%4 = 1$$

$$\text{hash}_1(\text{key}) = \text{key} \% n$$

$$\text{hash}_2(\text{key}) = \text{key} \% 2n$$

LINEAR HASHING

Split
Pointer



Find 6

$$\text{hash}_1(6) = 6\%4 = 2$$

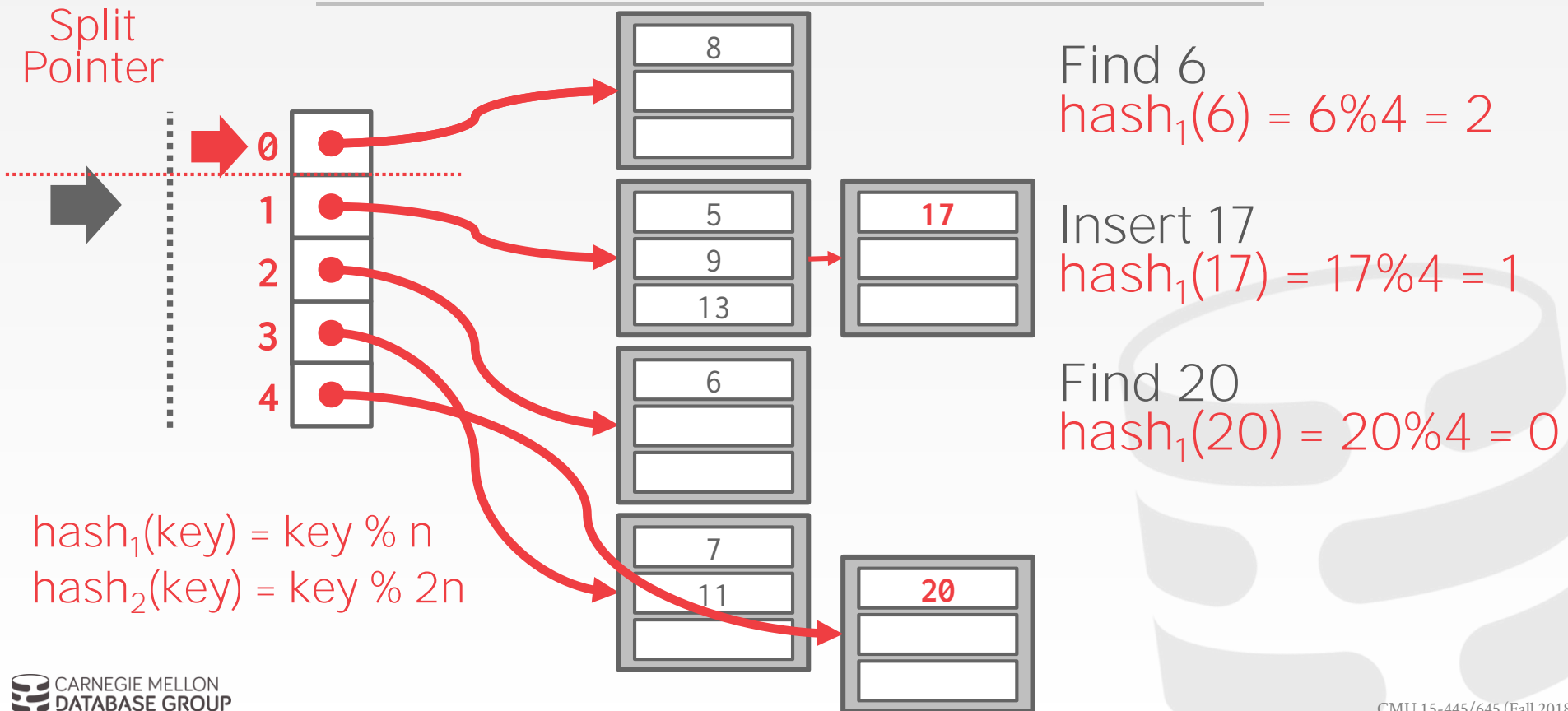
Insert 17

$$\text{hash}_1(17) = 17\%4 = 1$$

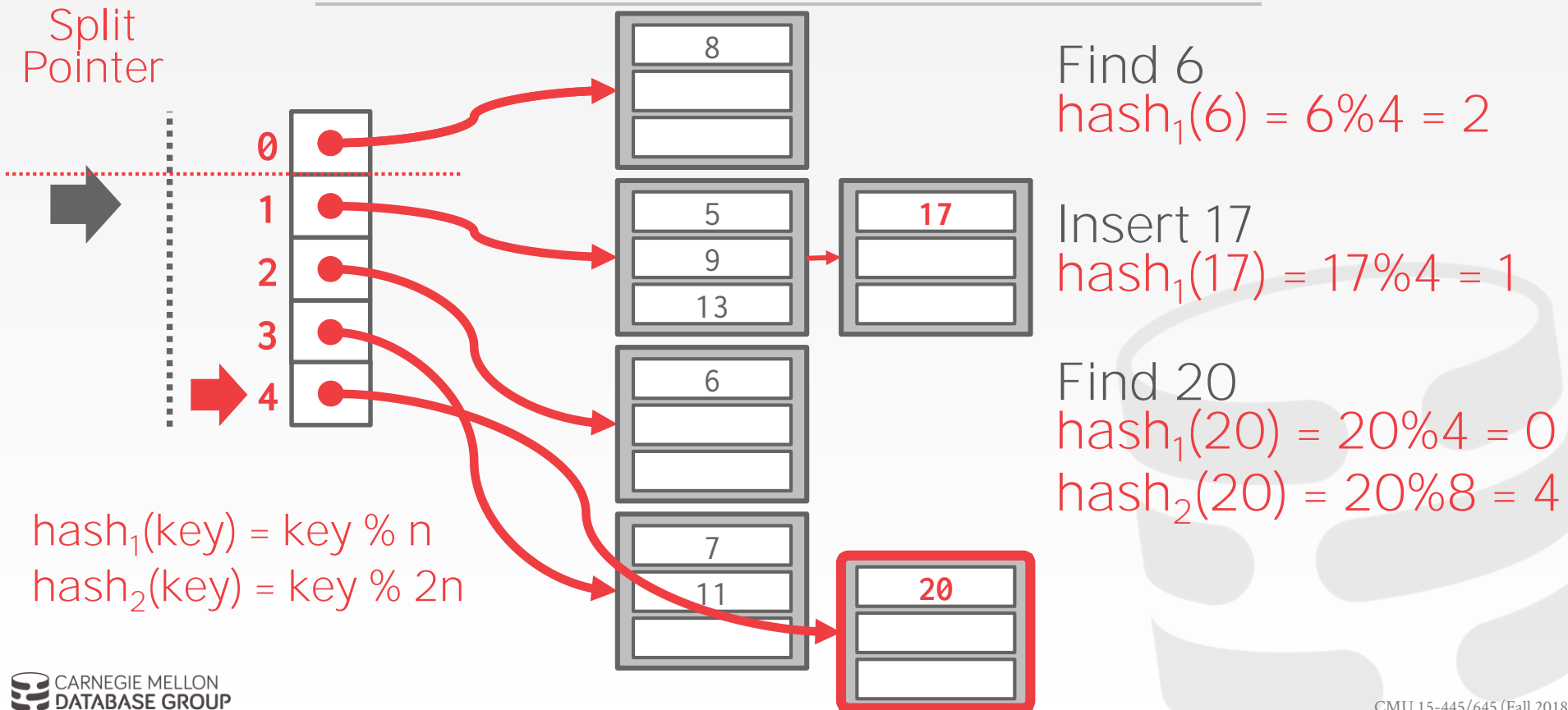
$$\text{hash}_1(\text{key}) = \text{key} \% n$$

$$\text{hash}_2(\text{key}) = \text{key} \% 2n$$

LINEAR HASHING



LINEAR HASHING



LINEAR HASHING

Splitting buckets based on the split pointer will eventually get to all overflowed buckets.

→ When the pointer reaches the last slot, delete the first hash function and move back to beginning.

The pointer can also move backwards when buckets are empty.



CONCLUSION

Fast data structures that support $O(1)$ look-ups that are used all throughout the DBMS internals.
→ Trade-off between speed and flexibility.

Hash tables are usually **not** what you want to use for a table index...

Postgres Demo



NEXT CLASS

B+ Trees

Skip Lists

Radix Trees

