Carnegie Mellon University

# Sorting & Aggregations

Lecture #11

Database Systems
15-445/15-645
Fall 2018


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# TODAY'S AGENDA

Sorting Algorithms

Aggregations

# WHY DO WE NEED TO SORT?

Tuples in a table have no specific order

But users often want to retrieve tuples in a specific order.

→ Trivial to support duplicate elimination (**DISTINCT**)
→ Bulk loading sorted tuples into a B+ tree index is faster
→ Aggregations (**GROUP BY**)

# SORTING ALGORITHMS

If data fits in memory, then we can use a standard sorting algorithm like quick-sort.

If data does not fit in memory, then we need to use a technique that is aware of the cost of writing data out to disk.

# EXTERNAL MERGE SORT

## Sorting Phase

→ Sort small chunks of data that fit in main-memory, and then write back the sorted data to a file on disk.

## Merge Phase

→ Combine sorted sub-files into a single larger file.

# OVERVIEW

We will start with a simple example of a 2-way external merge sort.

Files are broken up into ***N*** pages.

The DBMS has a finite number of ***B*** fixed-size buffers.

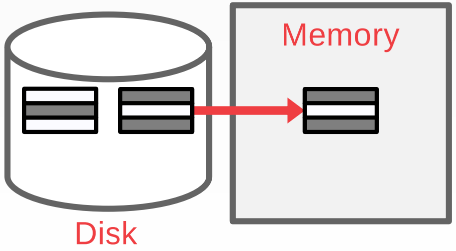
# 2-WAY EXTERNAL MERGE SORT

**Pass #0**

→ Reads every ***B*** pages of the table into memory
→ Sorts them, and writes them back to disk.
→ Each sorted set of pages is called a ***run***.

**Pass #1,2,3,…**

→ Recursively merges pairs of runs into runs twice as long
→ Uses three buffer pages (2 for input pages, 1 for output)



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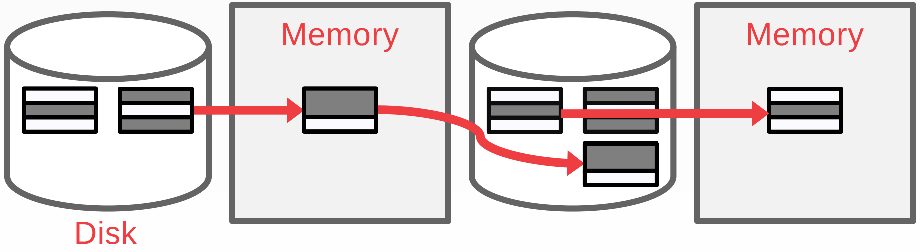

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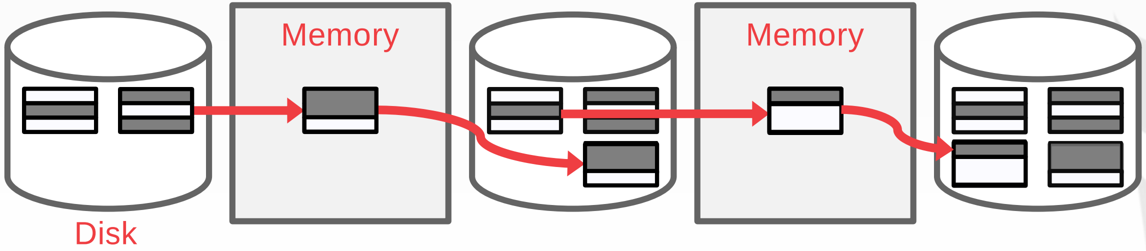

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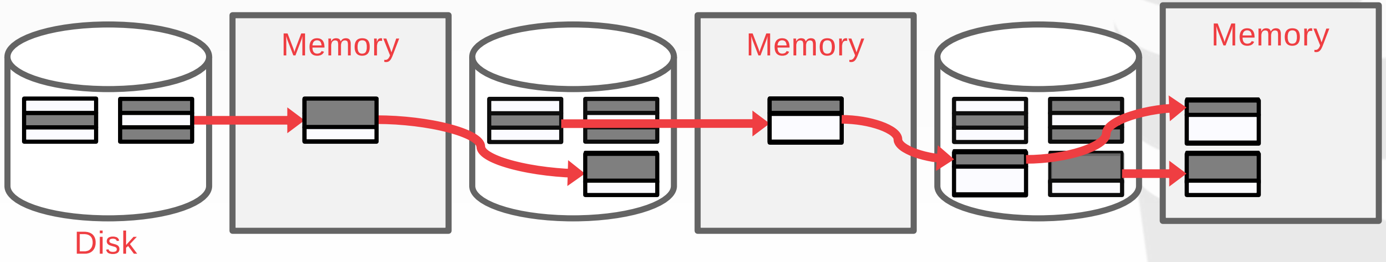

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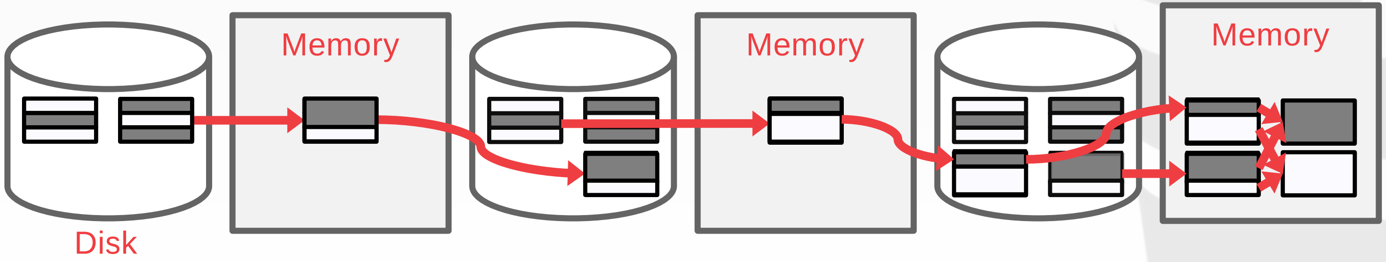

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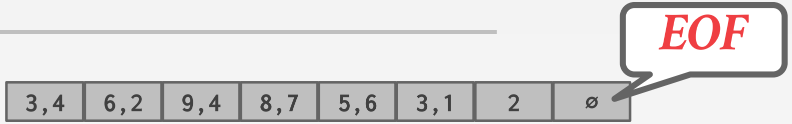


In each pass, we read and write each page in file.

Number of passes
$= \mathbf{1 + \lceil \log_2 \mathit{N} \rceil}$

Total I/O cost
$= \mathbf{2\mathit{N} \cdot (\#\ of\ passes)}$

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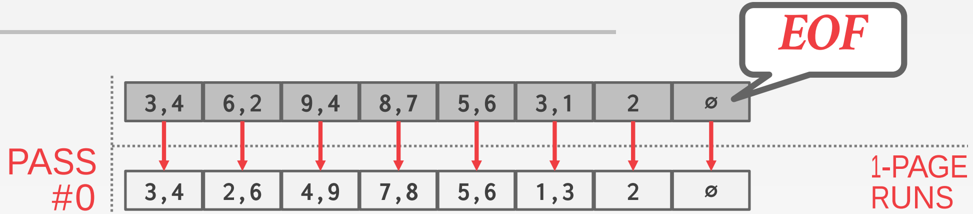

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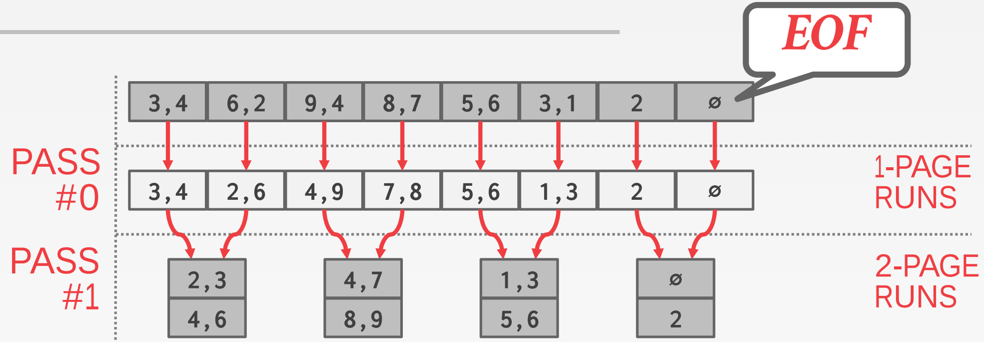

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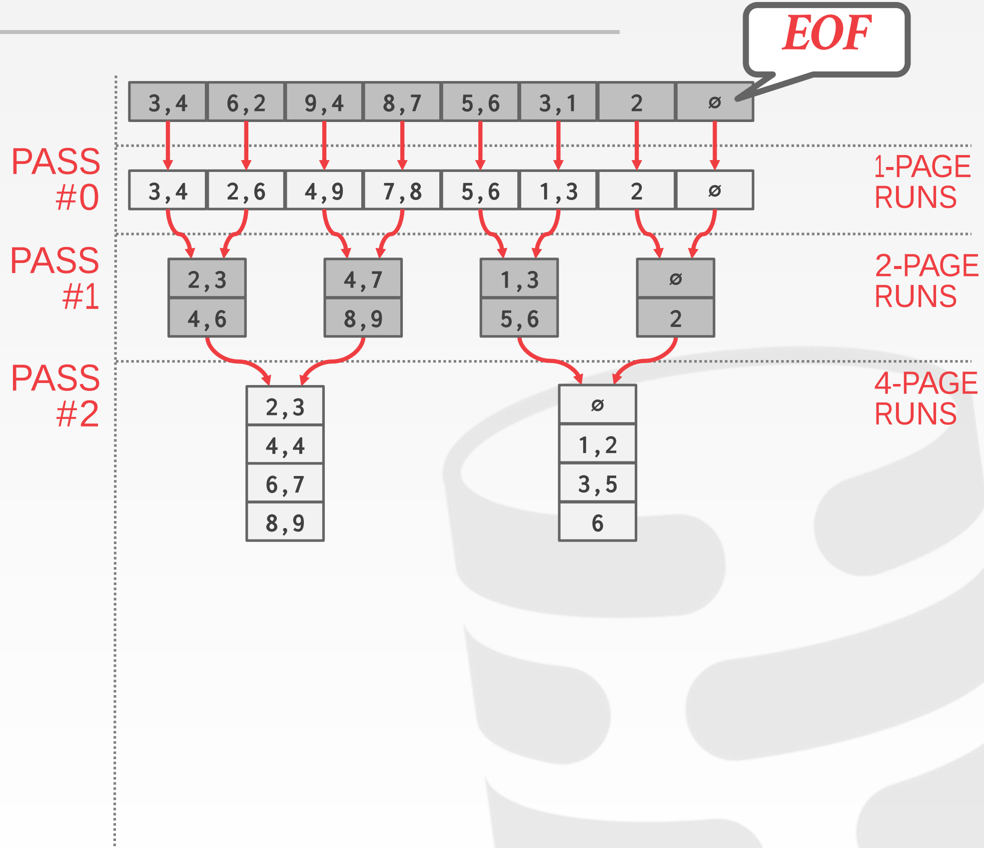

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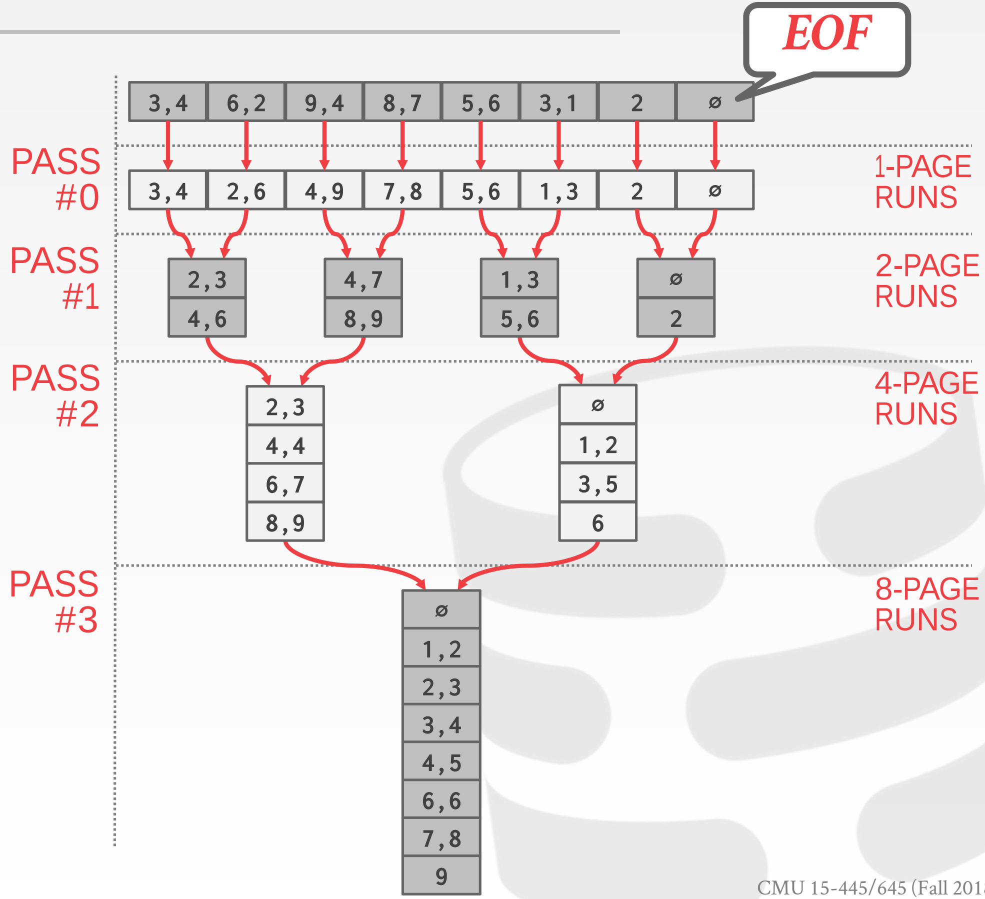

# 2-WAY EXTERNAL MERGE SORT

This algorithm only requires three buffer pages (***B*=3**).

Even if we have more buffer space available (***B*>3**), it does not effectively utilize them.

Let's next generalize the algorithm to make use of extra buffer space.

# GENERAL EXTERNAL MERGE SORT

**Pass #0**

→ Use $B$ buffer pages.

→ Produce $\lceil N / B \rceil$ sorted runs of size $B$

**Pass #1,2,3,…**

→ Merge $B$-1 runs (i.e., K-way merge).

Number of passes = $1 + \lceil \log_{B-1} \lceil N / B \rceil \rceil$

Total I/O Cost = $2N \cdot$ (# of passes)

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Total I/O Cost = $2N \cdot$ (# of passes)

# EXAMPLE

Sort 108 page file with 5 buffer pages: ***N*=108**, ***B*=5**

- → **Pass #0:** $\lceil N / B \rceil$ = $\lceil 108 / 5 \rceil$ = 22 sorted runs of 5 pages each (last run is only 3 pages)
- → **Pass #1:** $\lceil N' / B\text{-}1 \rceil$ = $\lceil 22 / 4 \rceil$ = 6 sorted runs of 20 pages each (last run is only 8 pages)
- → **Pass #2:** $\lceil N'' / B\text{-}1 \rceil$ = $\lceil 6 / 4 \rceil$ = 2 sorted runs, 80 pages and 28 pages
- → **Pass #3:** Sorted file of 108 pages

$$1+\lceil \log_{B-1}\lceil N / B \rceil \rceil = 1+\lceil \log_4 22 \rceil = 1+\lceil 2.229... \rceil$$

$$= \mathbf{4\ passes}$$

# USING B+TREES

If the table that must be sorted already has a B+ tree index on the sort attribute(s), then we can use that to accelerate sorting.

Retrieve tuples in desired sort order by simply traversing the leaf pages of the tree.

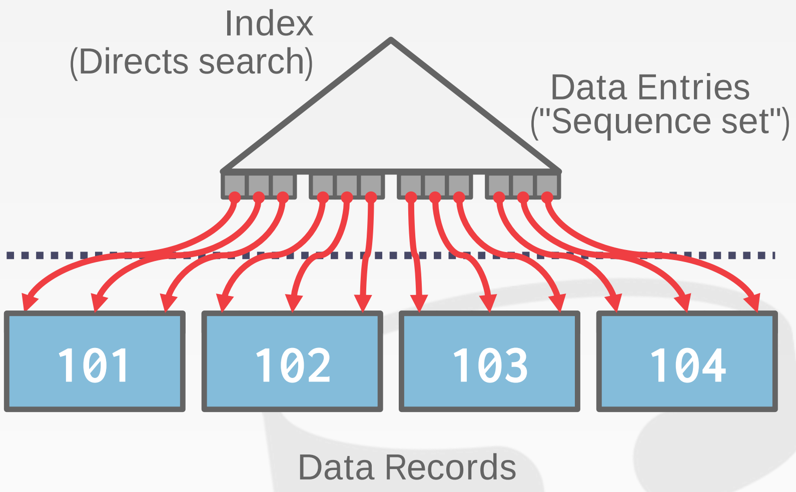
Cases to consider:

→ Clustered B+ tree
→ Unclustered B+ tree

# CASE 1: CLUSTERED B+TREE

Traverse to the left-most leaf page, and then retrieve tuples from all leaf pages.

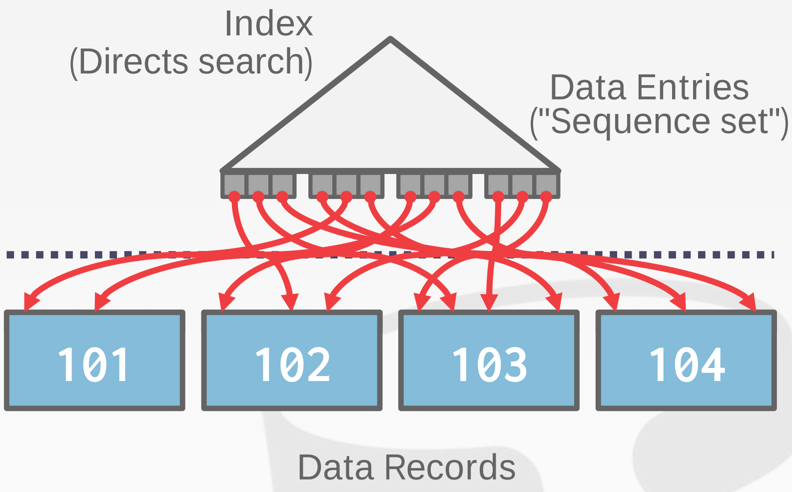
This will always better than external sorting.



# CASE 2: UNCLUSTERED B+TREE

Chase each pointer to the page that contains the data.

This is almost always a bad idea.
In general, one I/O per data record.



# AGGREGATIONS

Collapse multiple tuples into a single scalar value.

Two implementation choices:
→ Sorting
→ Hashing

# SORTING AGGREGATION

```
SELECT DISTINCT cid
  FROM enrolled
 WHERE grade IN ('B','C')
```

enrolled(sid,cid,grade)

| sid | cid | grade |
|---|---|---|
| 53666 | 15-445 | C |
| 53688 | 15-721 | A |
| 53688 | 15-826 | B |
| 53666 | 15-721 | C |
| 53655 | 15-445 | C |

Filter

| sid | cid | grade |
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Remove Columns

| cid |
|---|
| 15-445 |
| 15-826 |
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| 15-445 |

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Remove Columns

| cid |
|---|
| 15-445 |
| 15-826 |
| 15-721 |
| 15-445 |

Sort

| cid |
|---|
| 15-445 |
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| cid |
|---|
| 15-445 |
| 15-826 |
| 15-721 |
| 15-445 |

Sort

| cid |
|---|
| 15-445 |
| ~~15-445~~ |
| 15-721 |
| 15-826 |

Eliminate Dupes

# ALTERNATIVES TO SORTING

What if we don't need the data to be ordered?

→ Forming groups in **GROUP BY** (no ordering)

→ Removing duplicates in **DISTINCT** (no ordering)

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Hashing is a better alternative in this scenario.
→ Only need to remove duplicates, no need for ordering.
→ Can be computationally cheaper than sorting.

# HASHING AGGREGATE

Populate an ephemeral hash table as the DBMS scans the table. For each record, check whether there is already an entry in the hash table:

→ **DISTINCT**: Discard duplicate.
→ **GROUP BY**: Perform aggregate computation.

If everything fits in memory, then it's easy.
If we have to spill to disk, then we need to be smarter…

# HASHING AGGREGATE

**Partition Phase**

→ Divide tuples into buckets based on hash key.

**ReHash Phase**

→ Build in-memory hash table for each partition and compute the aggregation.

# HASHING AGGREGATE PHASE #1: PARTITION

Use a hash function $h_1$ to split tuples into partitions on disk.

→ We know that all matches live in the same partition.

→ Partitions are "spilled" to disk via output buffers.

Assume that we have ***B*** buffers.

# HASHING AGGREGATE PHASE #1: PARTITION

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Remove Columns

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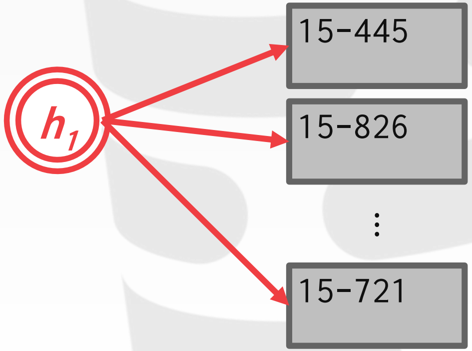
B-1 partitions



# HASHING AGGREGATE PHASE #2: REHASH

For each partition on disk:

→ Read it into memory and build an in-memory hash table based on a second hash function $h_2$.

→ Then go through each bucket of this hash table to bring together matching tuples.

This assumes that each partition fits in memory.

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Phase #1 Buckets

15-445

15-826

⋮

15-721

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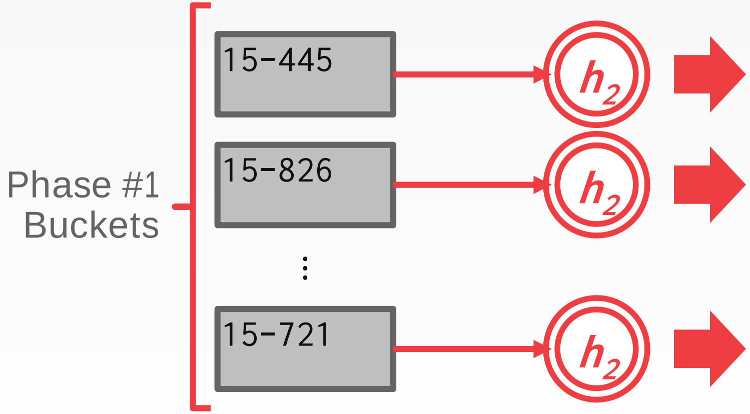


Hash Table

| Key | Value |
|---|---|
| XXX | 15-445 |
| YYY | 15-826 |
| ZZZ | 15-721 |

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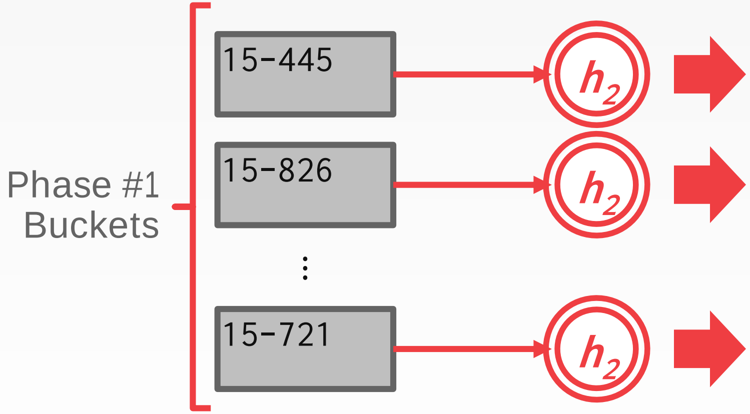


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| cid |
|---|
| 15-445 |
| 15-721 |
| 15-826 |

# HASHING SUMMARIZATION

During the ReHash phase, store pairs of the form (`GroupKey→RunningVal`)

When we want to insert a new tuple into the hash table:

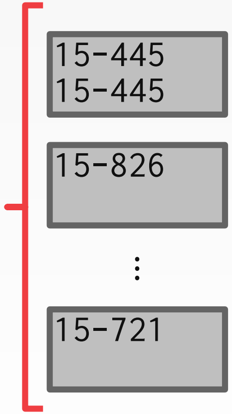
→ If we find a matching `GroupKey`, just update the `RunningVal` appropriately

→ Else insert a new `GroupKey→RunningVal`

# HASHING SUMMARIZATION

```sql
SELECT cid, AVG(s.gpa)
  FROM student AS s, enrolled AS e
 WHERE s.sid = e.sid
 GROUP BY cid
```

Phase #1
Buckets



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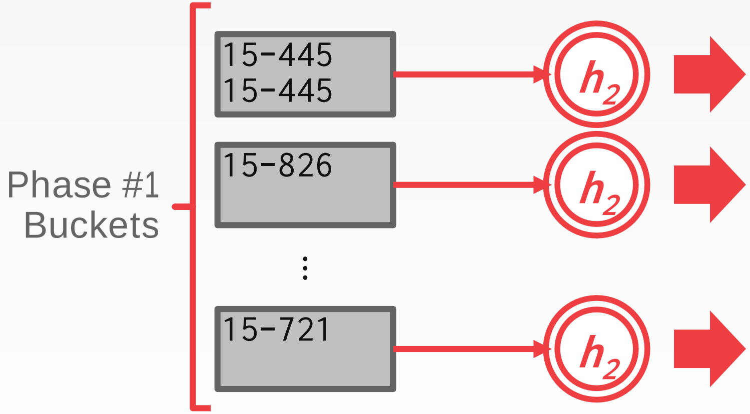


Hash Table

| key | value |
|---|---|
| XXX | 15-445→(2,7.32) |
| YYY | 15-826→(1,3.33) |
| ZZZ | 15-721→(1,2.89) |

# HASHING SUMMARIZATION

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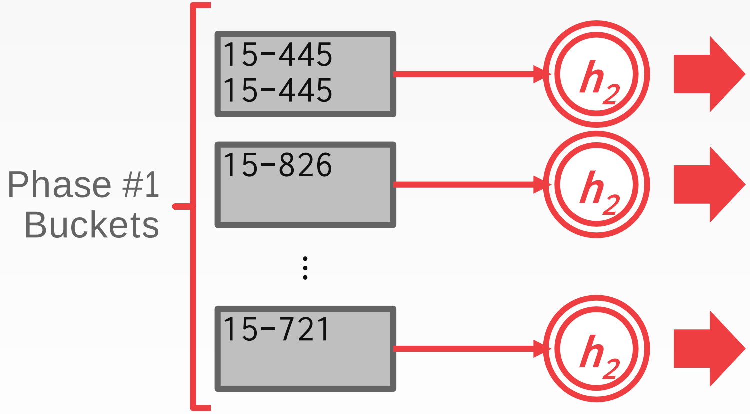
Running Totals

AVG(col) → (COUNT,SUM)
MIN(col) → (MIN)
MAX(col) → (MAX)
SUM(col) → (SUM)
COUNT(col) → (COUNT)

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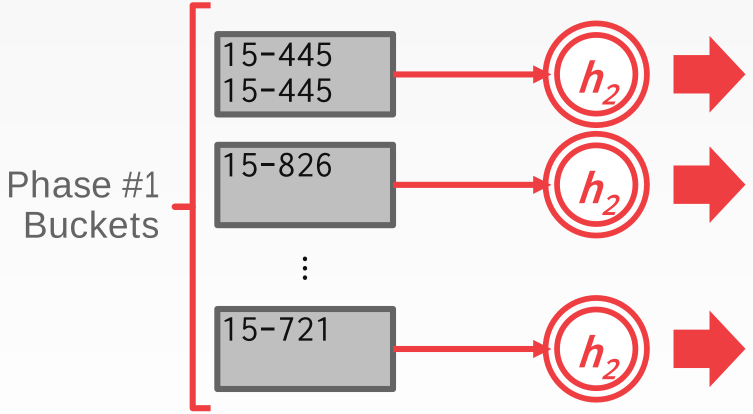


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## Final Result

| cid | AVG(gpa) |
|---|---|
| 15-445 | 3.66 |
| 15-826 | 3.33 |
| 15-721 | 2.89 |

# COST ANALYSIS

How big of a table can we hash using this approach?

→ ***B*-1** "spill partitions" in Phase #1

→ Each should be no more than ***B*** blocks big

Answer: $\boldsymbol{B \cdot (B\text{-}1)}$

→ A table of ***N*** pages needs about **sqrt(*N*)** buffers

→ Assumes hash distributes records evenly. Use a "fudge factor" ***f*>1** for that: we need $\boldsymbol{B \cdot \mathrm{sqrt}(f \cdot N)}$

# CONCLUSION

Choice of sorting vs. hashing is subtle and depends on optimizations done in each case.

We already discussed the optimizations for sorting:

→ Chunk I/O into large blocks to amortize seek+RD costs.
→ Double-buffering to overlap CPU and I/O.

# NEXT CLASS

Nested Loop Join

Sort-Merge Join

Hash Join

"Exotic" Joins