IMPORTANT:
- Upload this PDF with your answers to Gradescope by 11:59pm on Monday Sept 30, 2019.
- Plagiarism: Homework may be discussed with other students, but all homework is to be completed individually.

For your information:
- Graded out of 100 points; 4 questions total
- Rough time estimate: ≈1-4 hours (0.5-1 hours for each question)

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Number of Days this Assignment is Late: __________

Number of Late Day You Have Left: __________
Question 1: Cuckoo Hashing ........................................... [20 points]

Consider the following cuckoo hashing schema:

1. Both tables have a size of 4.
2. The hashing function of the first table returns the lowest two bits: \( h_1(x) = x \ & \ 0b11 \).
3. The hashing function of the second table returns the next two bits: \( h_2(x) = (x >> 2) \ & \ 0b11 \).
4. When replacement is necessary, first select an element in the second table.
5. The original content is shown in Figure 1.

<table>
<thead>
<tr>
<th>Table 1</th>
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<td>4</td>
<td></td>
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<td></td>
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Figure 1: Initial contents of the hash tables.

Use the following template to answer the questions: https://cmudb.io/fall2019-hw1.

(a) [4 points] Insert keys 12 and 10. Draw the resulting two tables.
(b) [4 points] Then delete 14, and insert 8. Draw the resulting two tables.
(c) [6 points] Finally, insert 28. Draw the resulting two tables.
(d) [6 points] What is the smallest key that potentially causes an infinite loop given the tables in (c)

- □ 0
- □ 2
- □ 5
- □ 6
- □ 7
- □ 9
- □ None of the above
Question 2: B+Tree .................................................. [45 points]

Consider the following B+ tree.

![B+ Tree](https://cmudb.io/fall2019-hw2)

Figure 2: B+ Tree of order $d = 4$ and height $h = 2$.

When answering the following questions, be sure to follow the procedures described in class and in your textbook. You can make the following assumptions:

- A left pointer in an internal node guides towards keys $<$ than its corresponding key, while a right pointer guides towards keys $\geq$.
- A leaf node underflows when the number of keys goes below $\lceil \frac{d-1}{2} \rceil$.
- An internal node underflows when the number of pointers goes below $\lceil \frac{d}{2} \rceil$.

Use the following draw.io template for your answers:
https://cmudb.io/fall2019-hw2

(a) [15 points] Insert $10^*$ into the B+tree. Draw the resulting tree.

(b) [5 points] How many pointers (parent-to-child and sibling-to-sibling) do you chase to find all keys between 5 and 15?

- [ ] 2
- [ ] 3
- [ ] 4
- [ ] 5
- [X] 6
- [X] 7

(c) [15 points] Then delete $23^*$. Draw the resulting tree.

(d) [10 points] Finally insert $4^*$ and delete $11^*$. Draw the resulting tree.

Homework 2 continues...
**Question 3: Extendible Hashing** ........................................... [25 points]

Consider an extendible hashing structure such that:

- Each bucket can hold up to two records.
- The hashing function uses the lowest $g$ bits, where $g$ is the global depth.

(a) Starting from an empty table, insert keys 15, 3, 7, 14.

- **[3 points]** What is the global depth of the resulting table?
  - [ ] 0  [ ] 1  [ ] 2  [ ] 3  [ ] 4  [ ] None of the above

- **[3 points]** What is the local depth the bucket containing 14?
  - [ ] 0  [ ] 1  [ ] 2  [ ] 3  [ ] 4  [ ] None of the above

- **[3 points]** What is the local depth of the bucket containing 3?
  - [ ] 0  [ ] 1  [ ] 2  [ ] 3  [ ] 4  [ ] None of the above

(b) Starting from the result in (a), you insert keys 1, 9, 23, 11, 17.

- **[4 points]** Which key will first cause a split (without doubling the size of the table)?
  - [ ] 1  [ ] 9  [ ] 23  [ ] 11  [ ] 17  [ ] None of the above

- **[4 points]** Which key will first make the table double in size?
  - [ ] 1  [ ] 9  [ ] 23  [ ] 11  [ ] 17  [ ] None of the above

(c) Now consider the table below, along with the following deletion rules:

1. If two buckets have the same local depth $d$, and share the first $d - 1$ bits of their indexes (e.g. 010 and 110 share the first 2 bits), then they can be merged if the total capacity fits in a single bucket. The resulting local depth is $d - 1$.
2. If the global depth $g$ becomes strictly greater than all local depths, then the table can be halved in size. The resulting global depth is $g - 1$.

![Figure 3: Extendible Hash Table along with the indexes of each bucket](image)

Question 3 continues…
Starting from the table above, delete keys 2, 7, 13, 15, 29.

i. [4 points] Which deletion first causes a reduction in a local depth.
   - 2  7  13  15  29  None of the above

ii. [4 points] Which deletion first causes a reduction in global depth.
    - 2  7  13  15  29  None of the above

Homework 2 continues...
Question 4: Suffix Trees ........................................ [10 points]

Consider the following suffix tree for unsigned 32-bit integers.

(a) [3 points] Which of the following elements belong to the suffix tree. Select all that apply.

☐ 0x45BD0000  ☐ 0x0000CAAC  ☐ 0xFFAAA00  ☐ 0xACCA0000  ☐ 0xBD000000

☐ None of the above

(b) [7 points] Insert the key 0x00FFAABB. Draw the resulting tree using this template: https://cmudb.io/fall2019-hw4.