CARNEGIE MELLON UNIVERSITY
COMPUTER SCIENCE DEPARTMENT
15-445/645 – DATABASE SYSTEMS (FALL 2020)
PROF. ANDY PAVLO

# Homework #2 (by Ian Romines)
Due: **Sunday October 4, 2020 @ 11:59pm**

**IMPORTANT:**

- **Upload this PDF** with your answers to **Gradescope by 11:59pm on Sunday October 4, 2020.**
- **Plagiarism**: Homework may be discussed with other students, but all homework is to be completed **individually**.
- **You have to use this PDF for all of your answers.**

For your information:

- Graded out of **100** points; **4** questions total
- Rough time estimate: ≈1-4 hours (0.5-1 hours for each question)

*Revision* : 2020/09/29 10:44

| Question | Points | Score |
|---|---|---|
| Cuckoo Hashing | 20 | |
| B+Tree | 45 | |
| Extendible Hashing | 25 | |
| Suffix Trees | 10 | |
| Total: | 100 | |

## Question 1: Cuckoo Hashing . . . . . [20 points]

Consider the following cuckoo hashing schema:

1. Both tables have a size of 4.
2. The hashing function of the first table returns the lowest two bits: $h_1(\texttt{x}) = \texttt{x \& 0b11}$.
3. The hashing function of the second table returns the next two bits: $h_2(\texttt{x}) = \texttt{(x >> 2) \& 0b11}$
4. When replacement is necessary, first select an element in the second table.
5. The original content is shown in Figure 1.

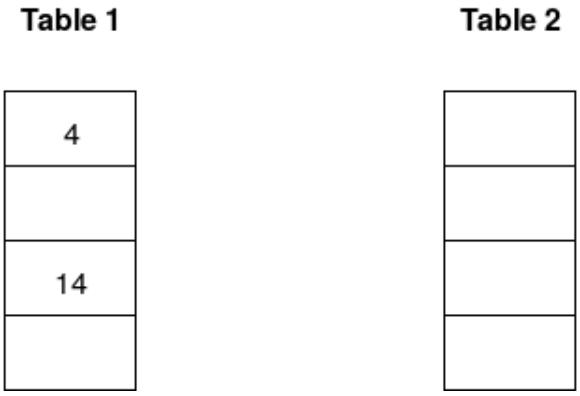
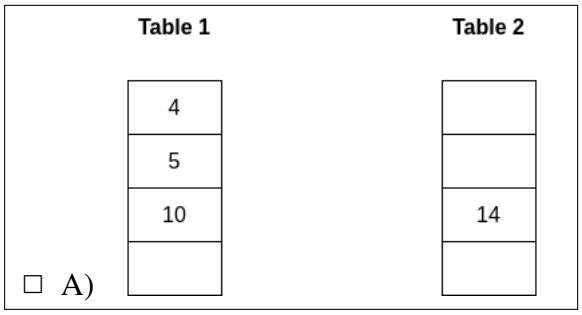
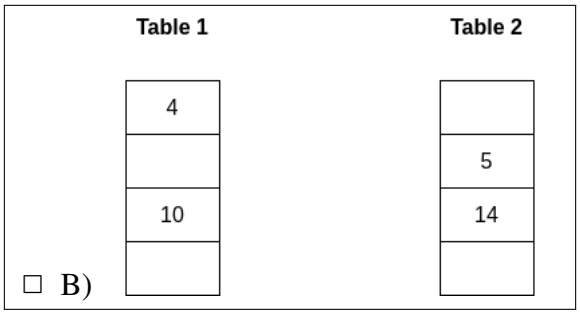


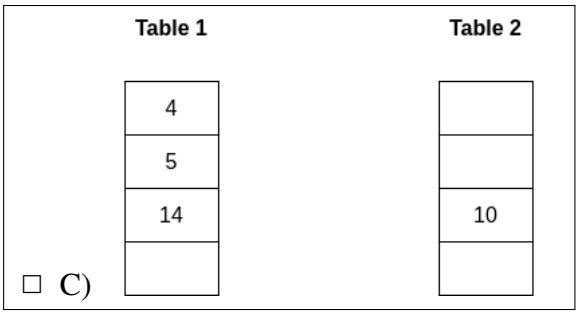

Figure 1: Initial contents of the hash tables.

(a) **[4 points]** Insert keys 5 and 10. Select the resulting two tables.

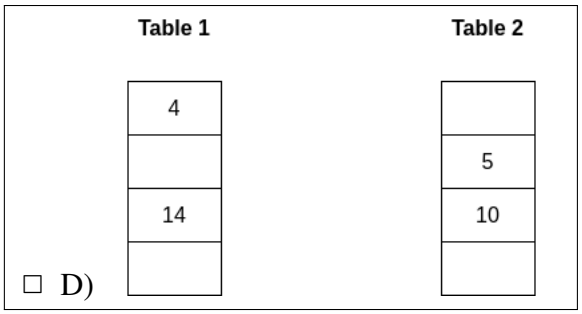


☐ C)

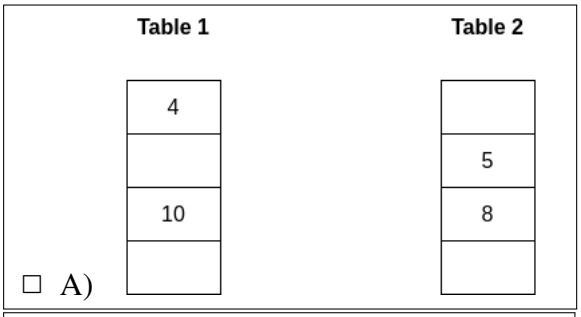



☐ D)

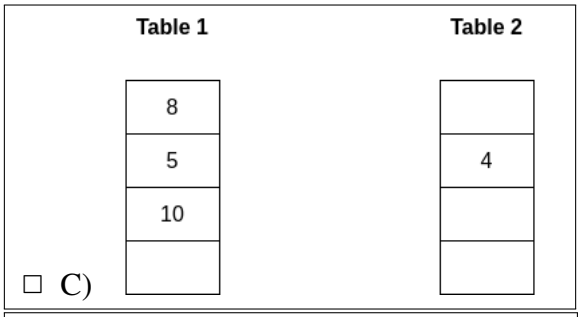



(b) **[4 points]** Then delete 14, and insert 8. Select the resulting two tables.

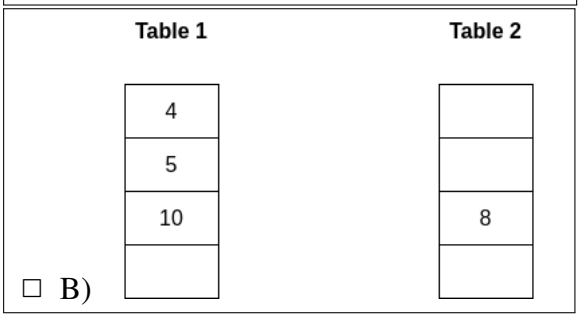
☐ A)




☐ C)




☐ B)

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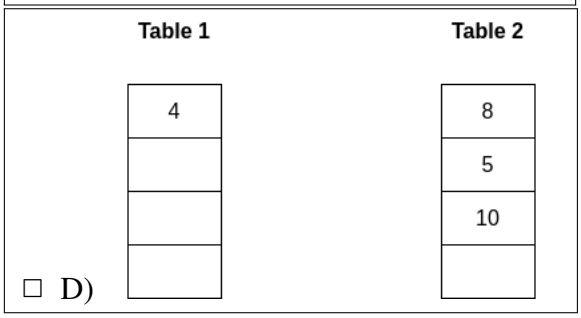
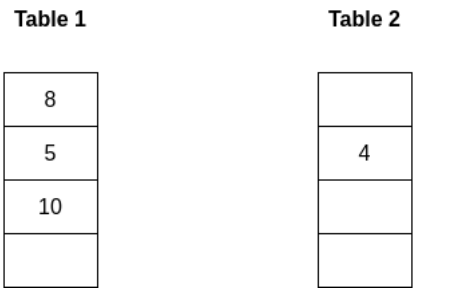
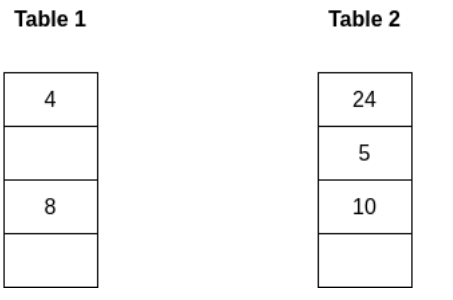


(c) **[6 points]** Finally, insert 24. Select the resulting two tables.

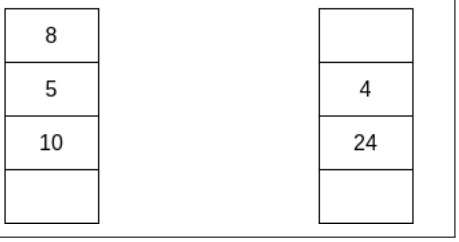
☐ A)




☐ C)




☐ B)

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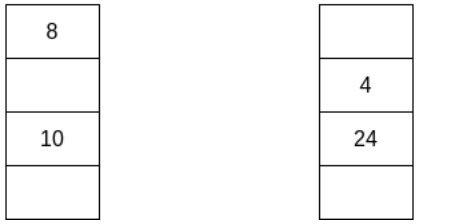



(d) **[6 points]** What is the smallest key that potentially causes an infinite loop given the tables in **(c)**

☐ 0 ☐ 2 ☐ 3 ☐ 6 ☐ 7 ☐ 9 ☐ None of the above

## Question 2: B+Tree . . . . . [45 points]

Consider the following B+tree.

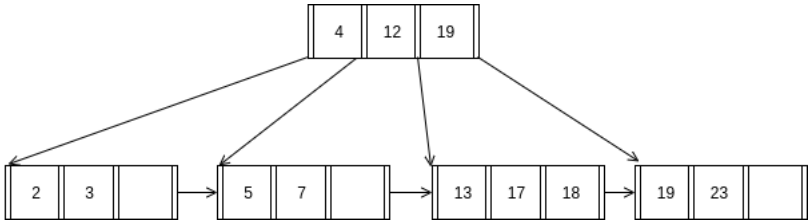


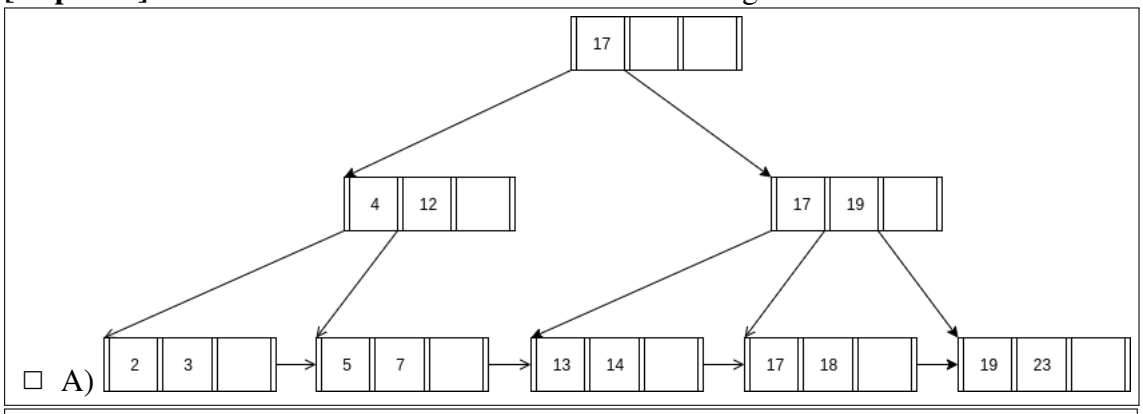

Figure 2: B+ Tree of order $d = 4$ and height $h = 2$.

When answering the following questions, be sure to follow the procedures described in class and in your textbook. You can make the following assumptions:

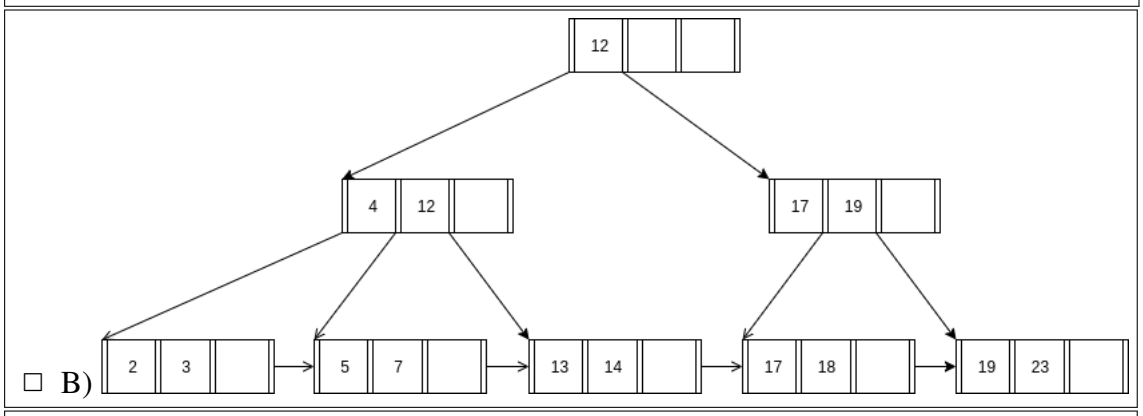
- A left pointer in an internal node guides towards keys $<$ than its corresponding key, while a right pointer guides towards keys $\geq$.
- A leaf node underflows when the number of **keys** goes below $\lceil \frac{d-1}{2} \rceil$.
- An internal node underflows when the number of **pointers** goes below $\lceil \frac{d}{2} \rceil$.

(a) **[15 points]** Insert $14^*$ into the B+tree. Select the resulting tree.

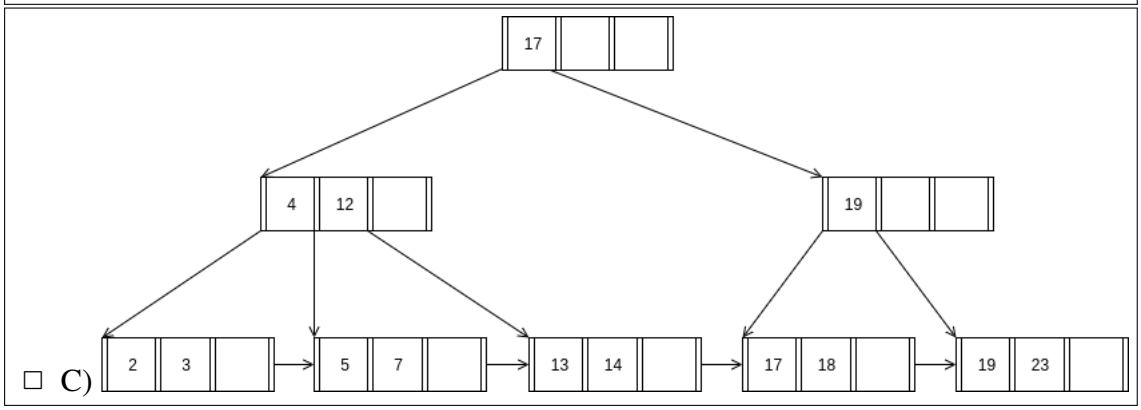
☐ A)



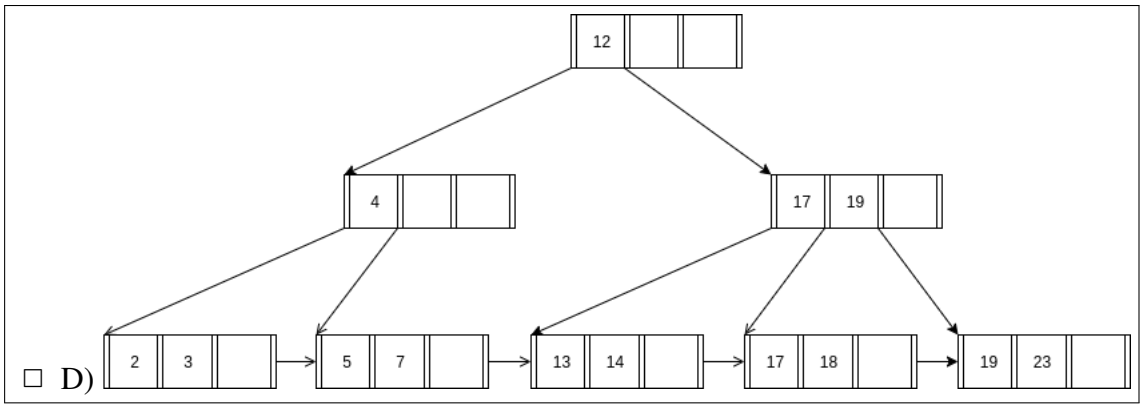

☐ B)




☐ C)



☐ D)

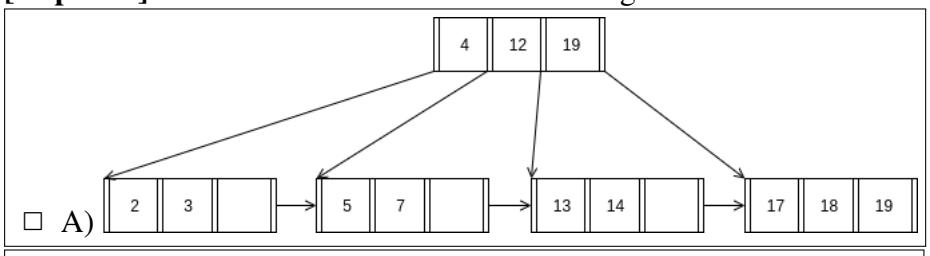



(b) **[5 points]** How many pointers (parent-to-child and sibling-to-sibling) do you chase to find all keys between 5 and 15?

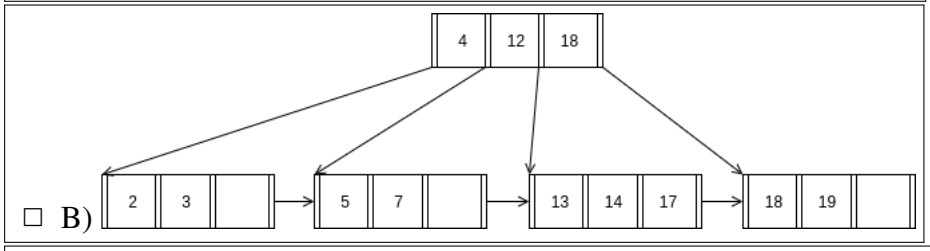
☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7

(c) **[15 points]** Then delete $23^*$. Select the resulting tree.

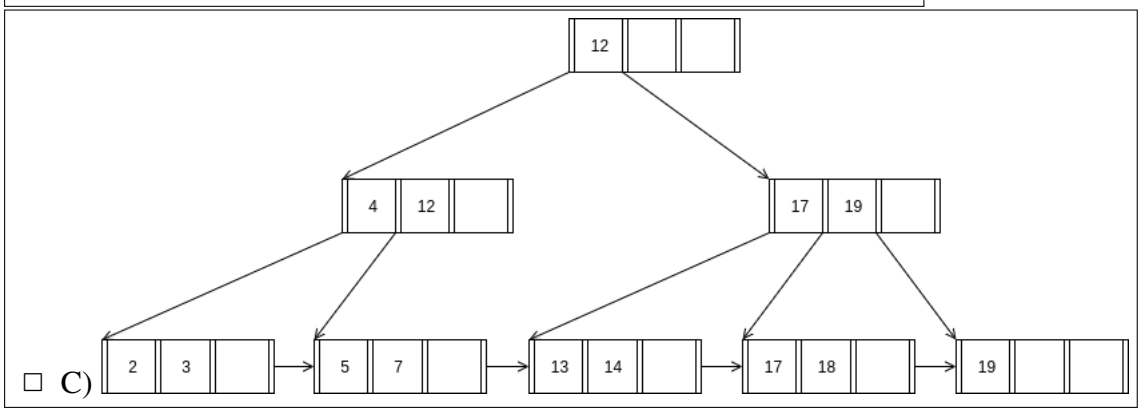
☐ A)

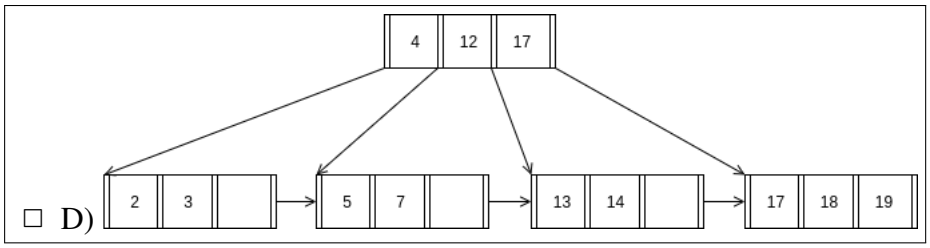



☐ B)



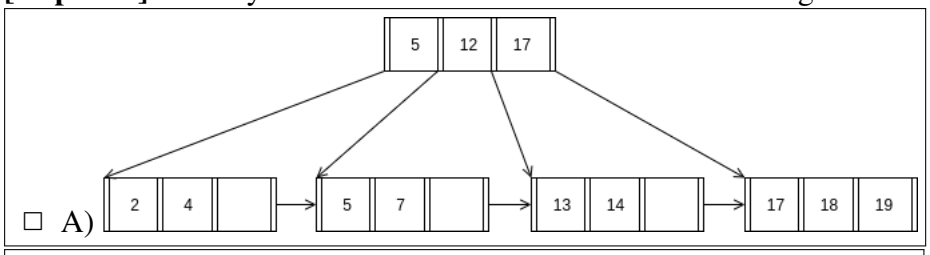

☐ C)



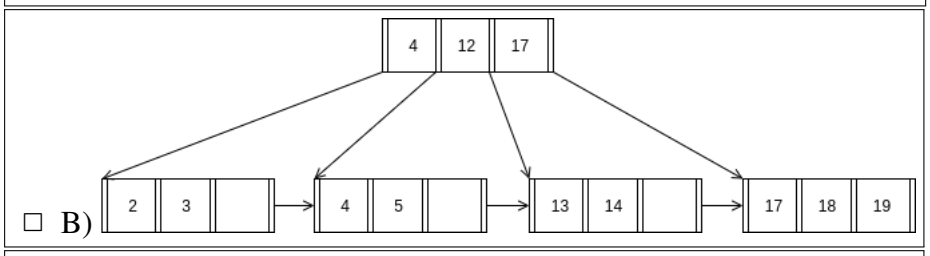



☐ D)

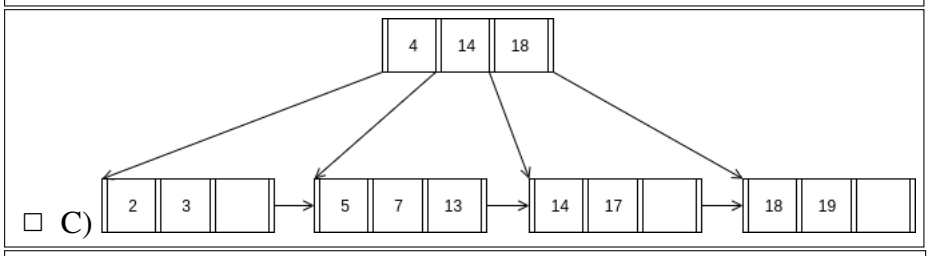
(d) **[10 points]** Finally insert $4^*$ and delete $3^*$. Select the resulting tree.



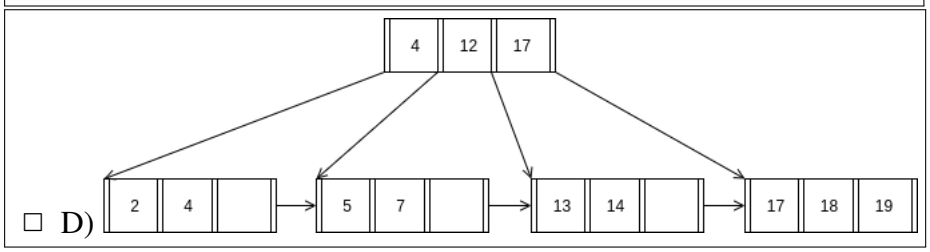

☐ A)




☐ B)




☐ C)




☐ D)

## Question 3: Extendible Hashing . . . . . [25 points]

Consider an extendible hashing structure such that:

- Each bucket can hold up to two records.
- The hashing function uses the lowest $g$ bits, where $g$ is the global depth.

(a) Starting from an empty table, insert keys 15, 2, 31, 11.

i. **[3 points]** What is the global depth of the resulting table?
☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ None of the above

ii. **[3 points]** What is the local depth the bucket containing 2?
☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ None of the above

iii. **[3 points]** What is the local depth of the bucket containing 31?
☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ None of the above

(b) Starting from the result in **(a)**, you insert keys 6, 9, 23, 12, 11.

i. **[4 points]** Which key will first cause a split (without doubling the size of the table)?
☐ 6 ☐ 9 ☐ 23 ☐ 12 ☐ 11 ☐ None of the above

ii. **[4 points]** Which key will first make the table double in size?
☐ 6 ☐ 9 ☐ 23 ☐ 12 ☐ 11 ☐ None of the above

(c) Now consider the table below, along with the following deletion rules:

1. If two buckets have the same local depth $d$, and share the first $d-1$ bits of their indexes (e.g. `010` and `110` share the first 2 bits), then they can be merged if the total capacity fits in a single bucket. The resulting local depth is $d-1$.
2. If the global depth $g$ becomes strictly greater than all local depths, then the table can be halved in size. The resulting global depth is $g-1$.

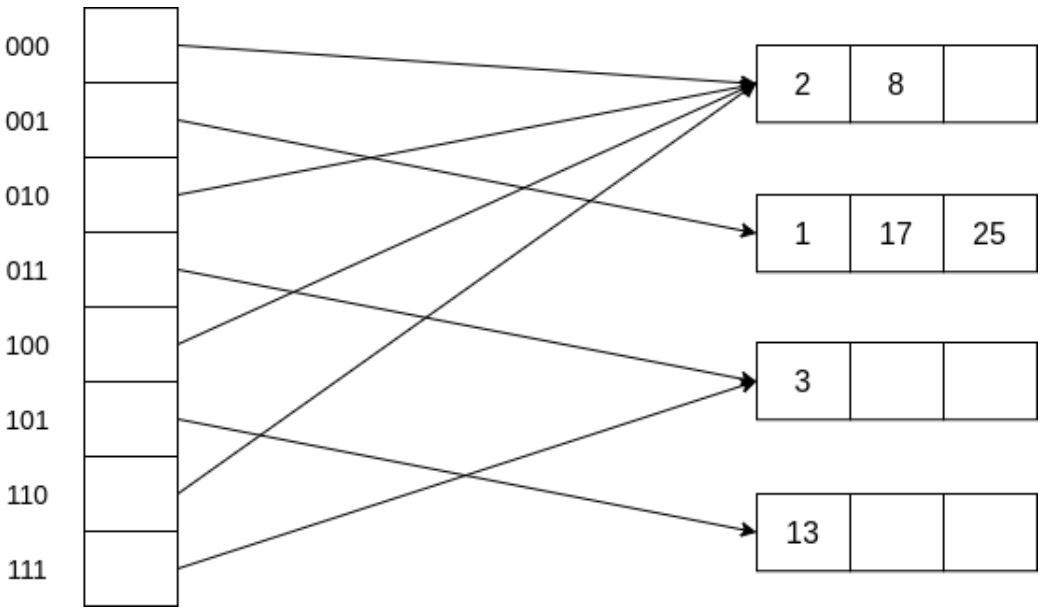



Figure 3: Extendible Hash Table along with the indexes of each bucket

Starting from the table above, delete keys 3, 8, 1, 2, 17.

i. **[4 points]** Which deletion first causes a reduction in a local depth.
☐ 3 ☐ 8 ☐ 1 ☐ 2 ☐ 17 ☐ None of the above

ii. **[4 points]** Which deletion first causes a reduction in global depth.
☐ 3 ☐ 8 ☐ 1 ☐ 2 ☐ 17 ☐ None of the above

## Question 4: Suffix Trees ..... [10 points]

Consider the following suffix tree for **unsigned 32-bit integers**.

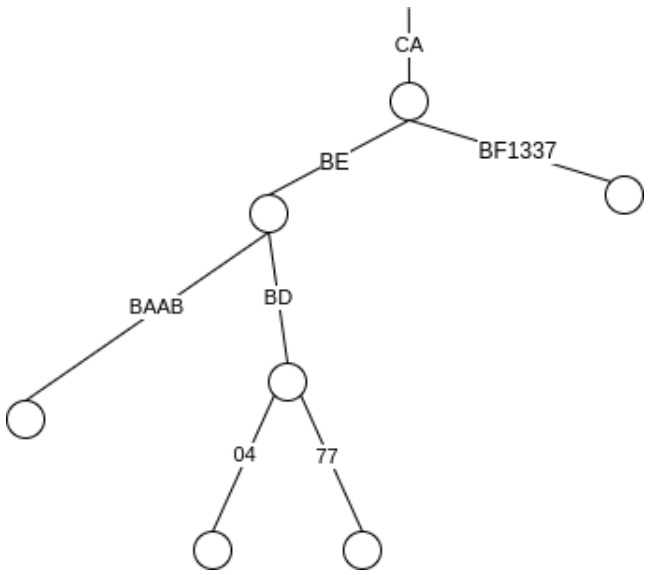


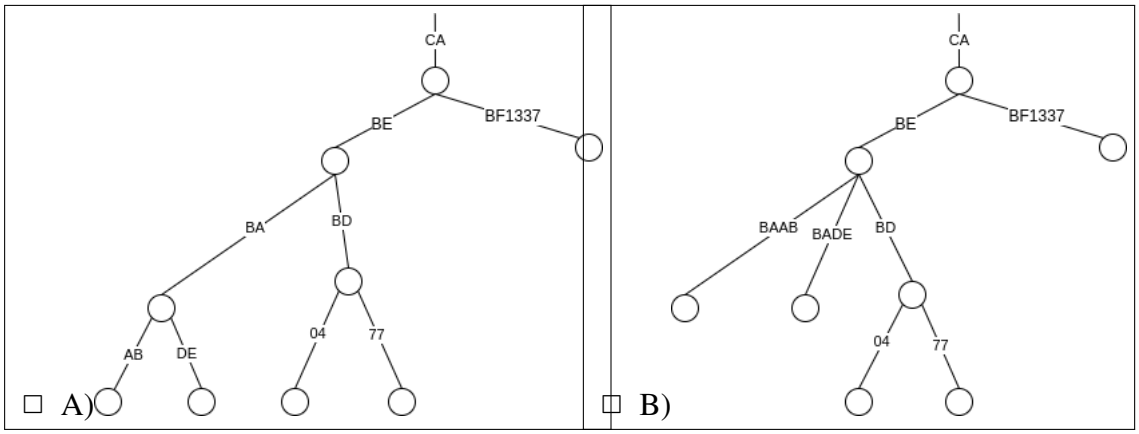

Figure 4: Suffix Tree

(a) **[3 points]** Which of the following elements belong to the suffix tree. Select all that apply.

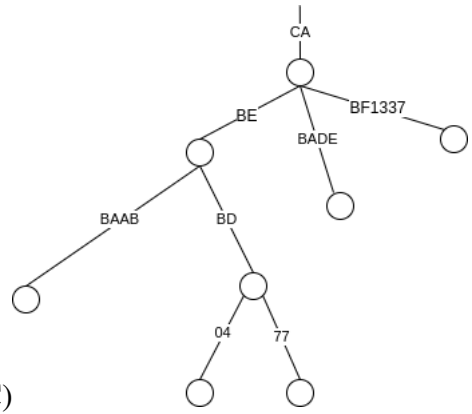
☐ `0xCABEACCA` ☐ `0xCA1337BF` ☐ `0x77BDBECA` ☐ `0xBAABCABE` ☐ `0xCABEBC04`

☐ None of the above

(b) **[7 points]** Insert the key `0xCABEBADE`. Select the resulting tree.



☐ C)




☐ D)