Carnegie Mellon University

Intro to Database Systems (15-445/645)

Lecture #08

B+Tree Index

FALL 2023 » Prof. Andy Pavlo • Prof. Jignesh Patel

UPCOMING EVENTS

PostgresML (ML⇄DB Seminar)
→ Monday Sept 25th @ 4:30pm

Weaviate (ML⇄DB Seminar)
→ Monday Oct 2nd @ 4:30pm

FeatureForm (ML⇄DB Seminar)
→ Monday Oct 9th @ 4:30pm

LAST CLASS

Hash tables are important data structures that are used all throughout a DBMS.

→ Space Complexity: **O(n)**
→ Average Time Complexity: **O(1)**

Static vs. Dynamic Hashing schemes

DBMSs use mostly hash tables for their internal data structures.

TODAY'S AGENDA

B+Tree Overview
Design Choices
Optimizations

B-TREE FAMILY

There is a specific data structure called a **B-Tree**.

People also use the term to generally refer to a class of balanced tree data structures:

→ **B-Tree** (1971)
→ **B+Tree** (1973)
→ **B*Tree** (1977?)
→ **B$^{\text{link}}$-Tree** (1981)
→ **Bε-Tree** (2003)
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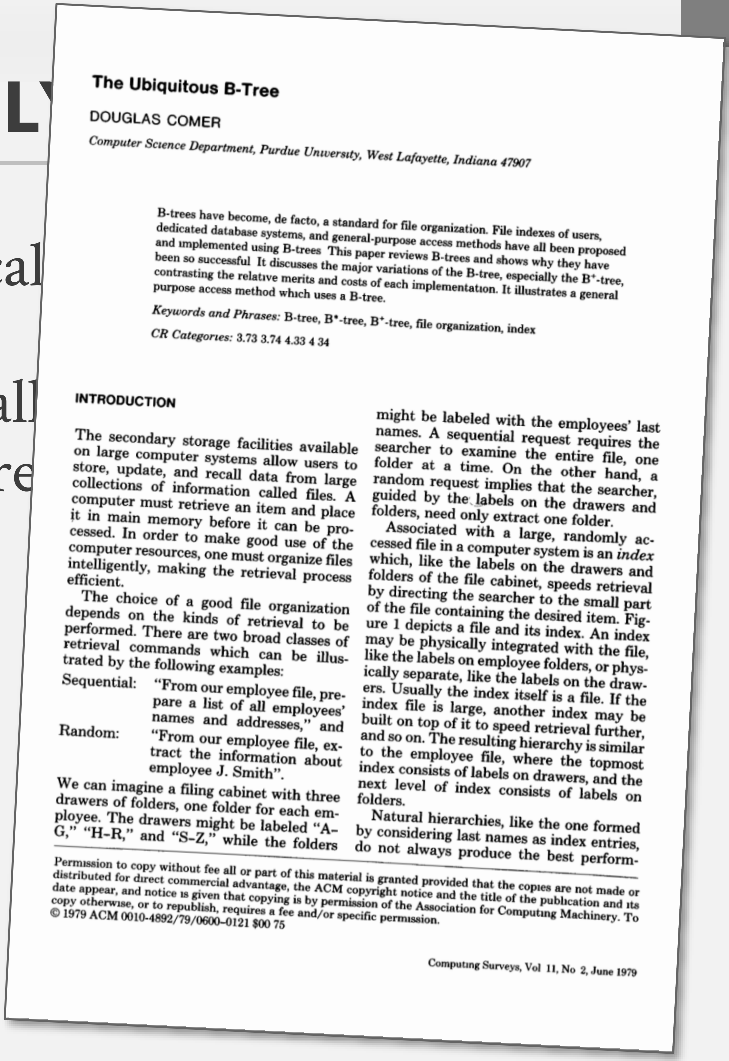

The Ubiquitous B-Tree

DOUGLAS COMER

Computer Science Department, Purdue University, West Lafayette, Indiana 47907

B-trees have become, de facto, a standard for file organization. File indexes of users, dedicated database systems, and general-purpose access methods have all been proposed and implemented using B-trees This paper reviews B-trees and shows why they have been so successful It discusses the major variations of the B-tree, especially the B⁺-tree, contrasting the relative merits and costs of each implementation. It illustrates a general purpose access method which uses a B-tree.

Keywords and Phrases: B-tree, B*-tree, B⁺-tree, file organization, index

CR Categories: 3.73 3.74 4.33 4 34

INTRODUCTION

The secondary storage facilities available on large computer systems allow users to store, update, and recall data from large collections of information called files. A computer must retrieve an item and place it in main memory before it can be processed. In order to make good use of the computer resources, one must organize files intelligently, making the retrieval process efficient.

The choice of a good file organization depends on the kinds of retrieval to be performed. There are two broad classes of retrieval commands which can be illustrated by the following examples:

Sequential: "From our employee file, prepare a list of all employees' names and addresses," and

Random: "From our employee file, extract the information about employee J. Smith".

We can imagine a filing cabinet with three drawers of folders, one folder for each employee. The drawers might be labeled "A–G," "H–R," and "S–Z," while the folders might be labeled with the employees' last names. A sequential request requires the searcher to examine the entire file, one folder at a time. On the other hand, a random request implies that the searcher, guided by the labels on the drawers and folders, need only extract one folder.

Associated with a large, randomly accessed file in a computer system is an *index* which, like the labels on the drawers and folders of the file cabinet, speeds retrieval by directing the searcher to the small part of the file containing the desired item. Figure 1 depicts a file and its index. An index may be physically integrated with the file, like the labels on employee folders, or physically separate, like the labels on the drawers. Usually the index itself is a file. If the index file is large, another index may be built on top of it to speed retrieval further, and so on. The resulting hierarchy is similar to the employee file, where the topmost index consists of labels on drawers, and the next level of index consists of labels on folders.

Natural hierarchies, like the one formed by considering last names as index entries, do not always produce the best perform-

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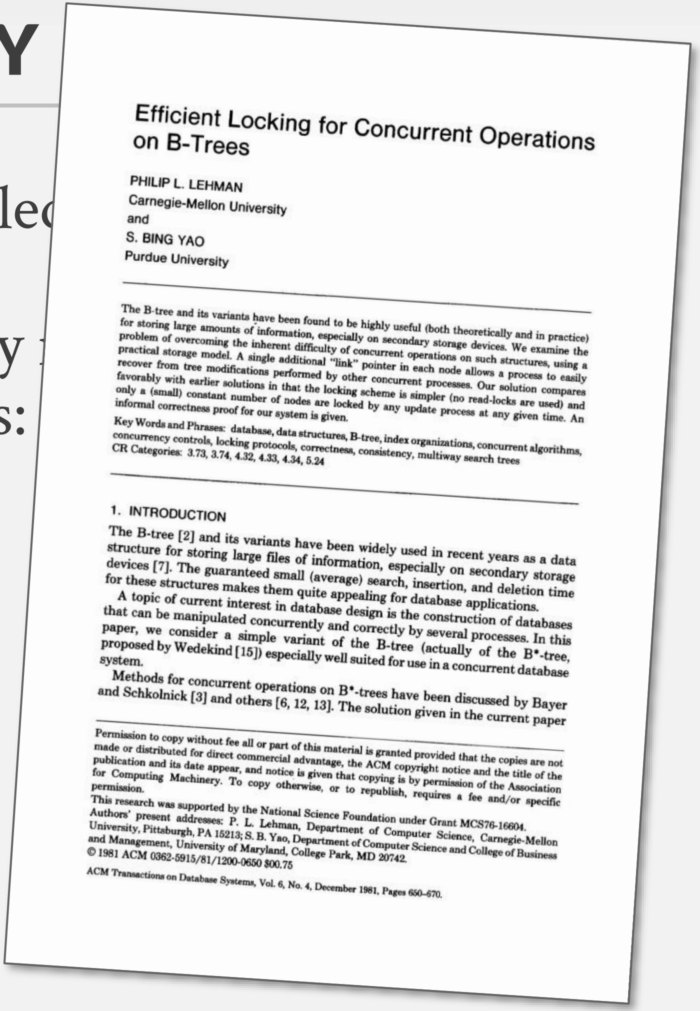

Efficient Locking for Concurrent Operations on B-Trees

PHILIP L. LEHMAN
Carnegie-Mellon University
and
S. BING YAO
Purdue University

The B-tree and its variants have been found to be highly useful (both theoretically and in practice) for storing large amounts of information, especially on secondary storage devices. We examine the problem of overcoming the inherent difficulty of concurrent operations on such structures, using a practical storage model. A single additional "link" pointer in each node allows a process to easily recover from tree modifications performed by other concurrent processes. Our solution compares favorably with earlier solutions in that the locking scheme is simpler (no read-locks are used) and only a (small) constant number of nodes are locked by any update process at any given time. An informal correctness proof for our system is given.

Key Words and Phrases: database, data structures, B-tree, index organizations, concurrent algorithms, concurrency controls, locking protocols, correctness, consistency, multiway search trees
CR Categories: 3.73, 3.74, 4.32, 4.33, 4.34, 5.24

1. INTRODUCTION

The B-tree [2] and its variants have been widely used in recent years as a data structure for storing large files of information, especially on secondary storage devices [7]. The guaranteed small (average) search, insertion, and deletion time for these structures makes them quite appealing for database applications.

A topic of current interest in database design is the construction of databases that can be manipulated concurrently and correctly by several processes. In this paper, we consider a simple variant of the B-tree (actually of the B*-tree, proposed by Wedekind [15]) especially well suited for use in a concurrent database system.

Methods for concurrent operations on B*-trees have been discussed by Bayer and Schkolnick [3] and others [6, 12, 13]. The solution given in the current paper

This research was supported by the National Science Foundation under Grant MCS76-16604.
Authors' present addresses: P. L. Lehman, Department of Computer Science, Carnegie-Mellon University, Pittsburgh, PA 15213; S. B. Yao, Department of Computer Science and College of Business and Management, University of Maryland, College Park, MD 20742.

postgres / src / backend / access / nbtree / **README**

1083 lines (959 loc) · 62.8 KB

Code | Blame | Raw

```
src/backend/access/nbtree/README

Btree Indexing
==============

This directory contains a correct implementation of Lehman and Yao's
high-concurrency B-tree management algorithm (P. Lehman and S. Yao,
Efficient Locking for Concurrent Operations on B-Trees, ACM Transactions
on Database Systems, Vol 6, No. 4, December 1981, pp 650-670).  We also
use a simplified version of the deletion logic described in Lanin and
Shasha (V. Lanin and D. Shasha, A Symmetric Concurrent B-Tree Algorithm,
Proceedings of 1986 Fall Joint Computer Conference, pp 380-389).

The basic Lehman & Yao Algorithm
--------------------------------
```

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B-TREE FAMILY

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B+TREE

A **B+Tree** is a self-balancing, ordered tree data structure that allows searches, sequential access, insertions, and deletions in **O(log n)**.

→ Generalization of a binary search tree, since a node can have more than two children.
→ Optimized for systems that read and write large blocks of data.

B+TREE PROPERTIES

A B+Tree is an ***M***-way search tree with the following properties:

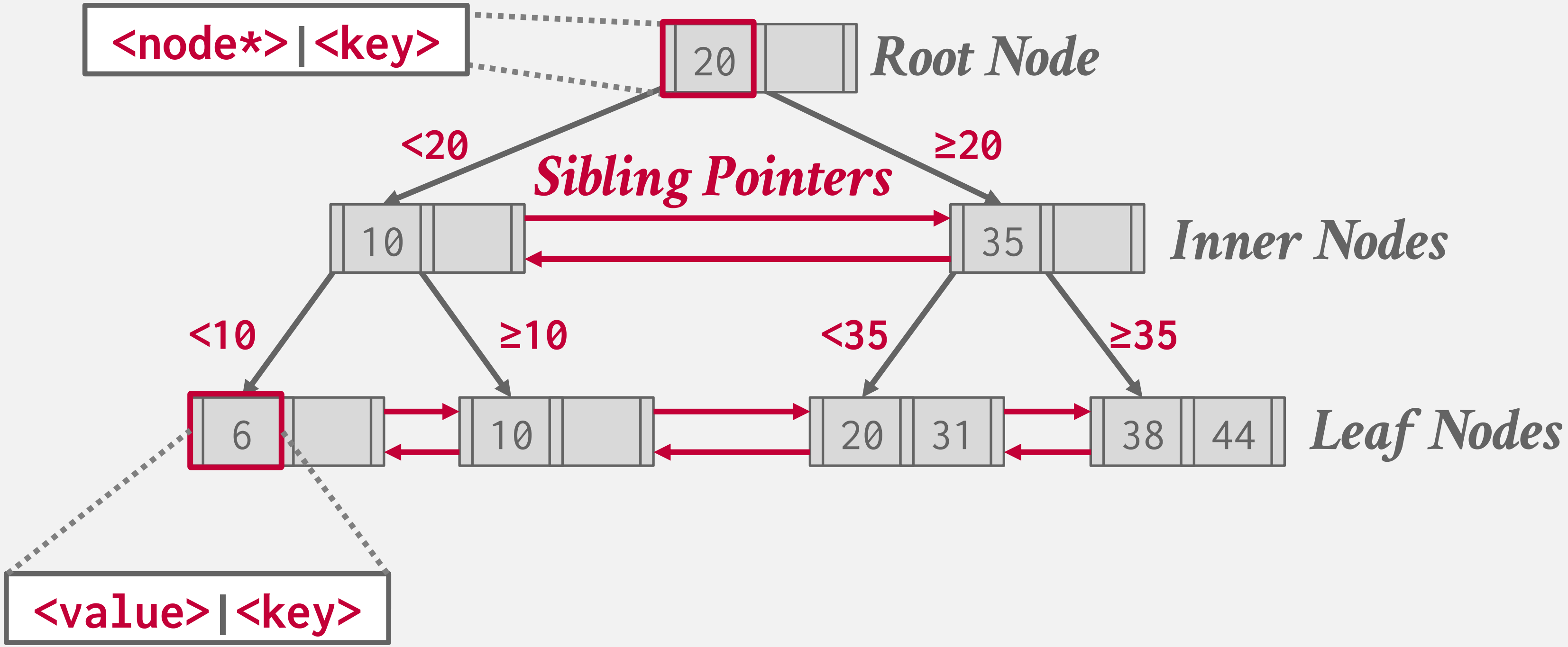
→ It is perfectly balanced (i.e., every leaf node is at the same depth in the tree)

→ Every node other than the root is at least half-full

`M/2-1 ≤ #keys ≤ M-1`

→ Every inner node with **`k`** keys has **`k+1`** non-null children

B+TREE EXAMPLE



NODES

Every B+Tree node is comprised of an array of key/value pairs.

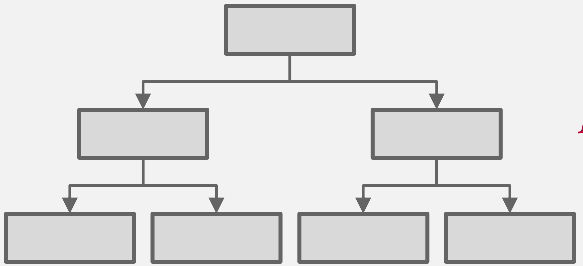
→ The keys are derived from the attribute(s) that the index is based on.

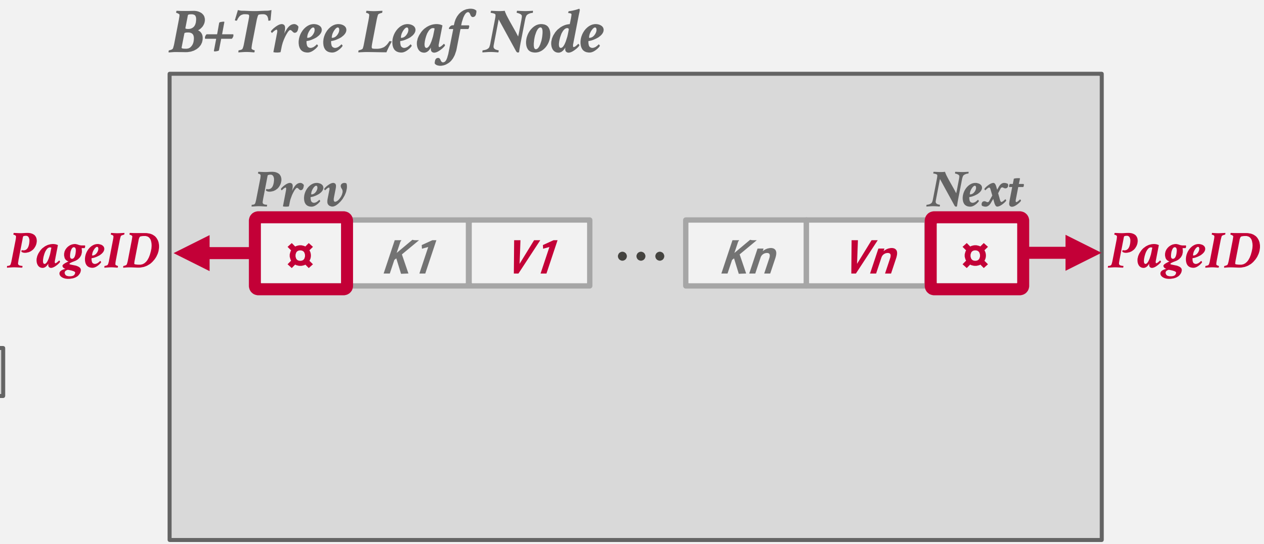
→ The values will differ based on whether the node is classified as an **inner node** or a **leaf node.**

The arrays are (usually) kept in sorted key order.

Store all NULL keys at either first or last leaf nodes.

B+TREE LEAF NODES





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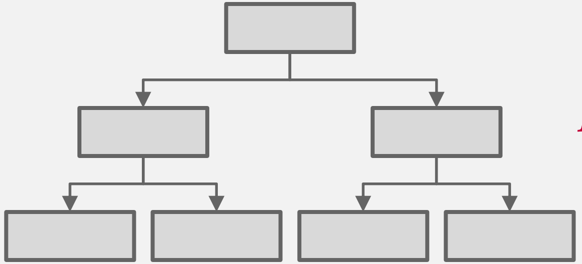

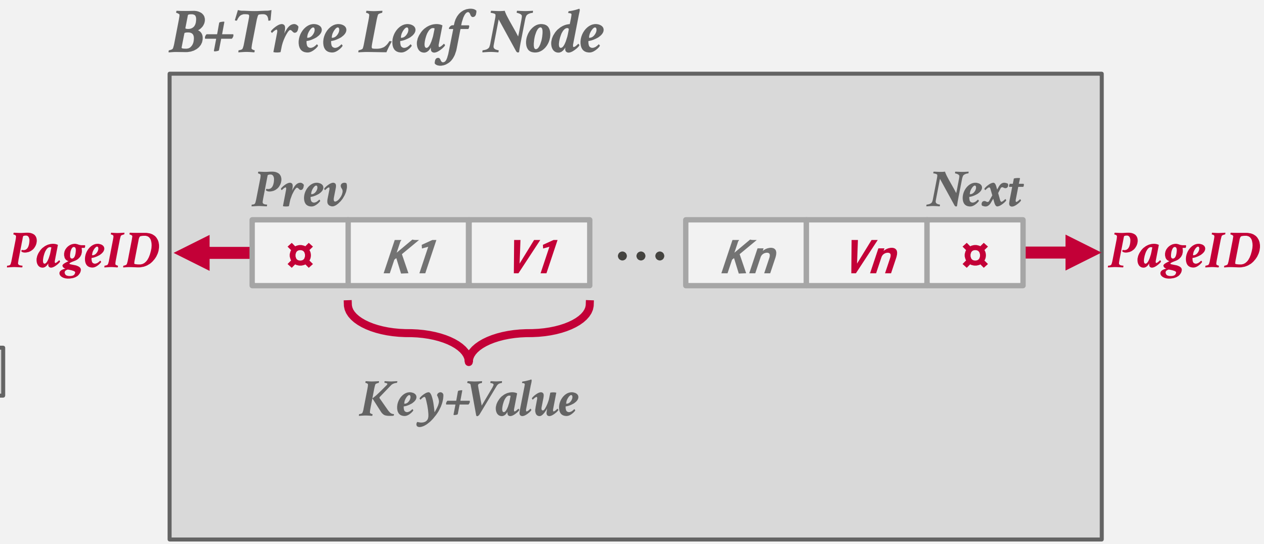

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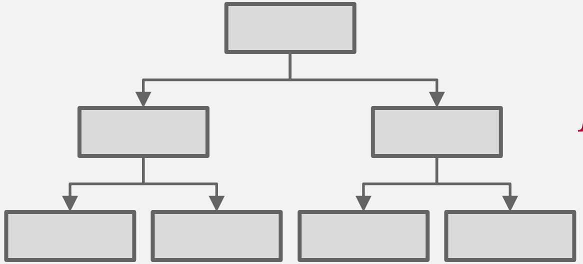

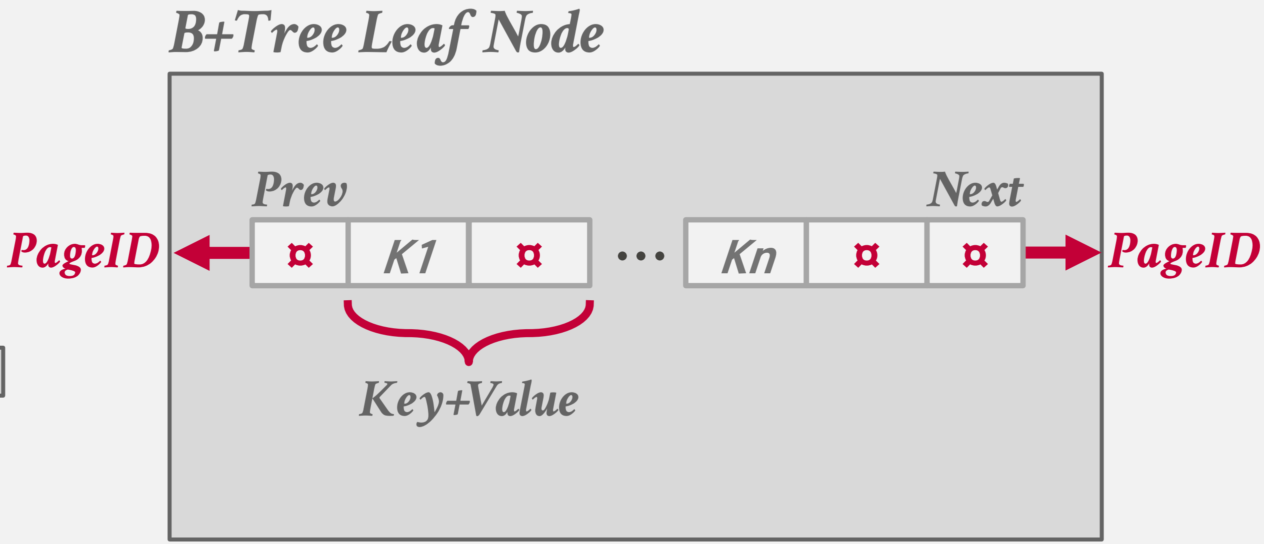

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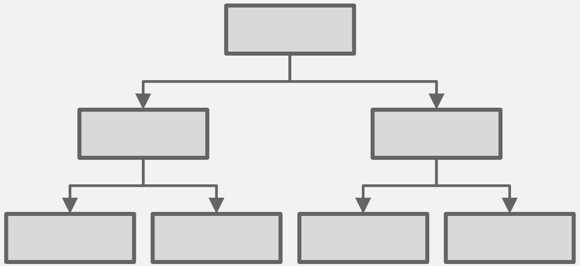
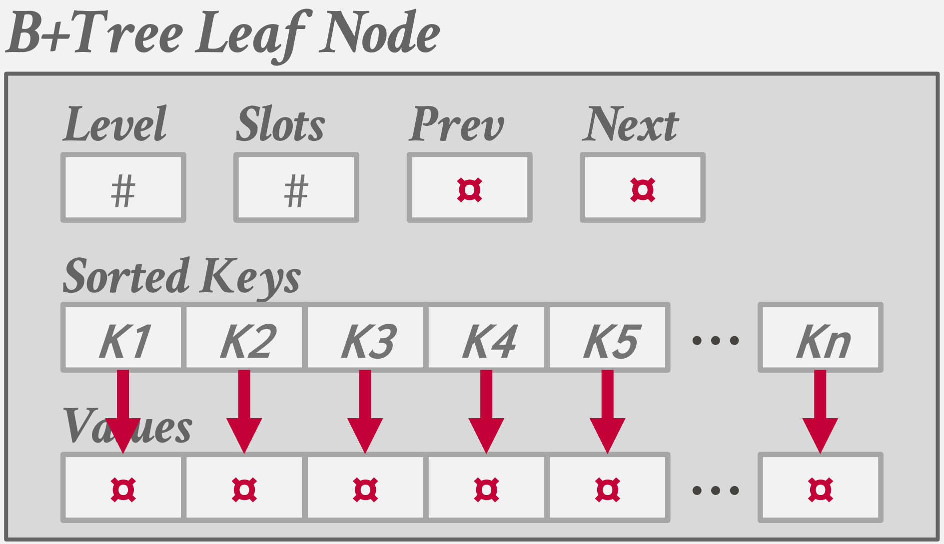




LEAF NODE VALUES

Approach #1: Record IDs

→ A pointer to the location of the tuple to which the index entry corresponds.

Approach #2: Tuple Data

→ AKA Index-Organized Storage

→ The leaf nodes store the actual contents of the tuple.

→ Secondary indexes must store the Record ID as their values.

B-TREE VS. B+TREE

The original **B-Tree** from 1972 stored keys and values in all nodes in the tree.

→ More space-efficient, since each key only appears once in the tree.

A **B+Tree** only stores values in leaf nodes. Inner nodes only guide the search process.

B+TREE – INSERT

Find correct leaf node **L**.
Insert data entry into **L** in sorted order.

If **L** has enough space, done!

Otherwise, split **L** keys into **L** and a new node **L2**
→ Redistribute entries evenly, copy up middle key.
→ Insert index entry pointing to **L2** into parent of **L**.

To split inner node, redistribute entries evenly, but push up middle key.

Source: Chris Re

B+TREE VISUALIZATION

https://cmudb.io/btree

Source: David Gales (Univ. of San Francisco)

B+TREE – DELETE

Start at root, find leaf **L** where entry belongs.
Remove the entry.
If **L** is at least half-full, done!
If **L** has only **M/2-1** entries,

→ Try to re-distribute, borrowing from sibling (adjacent node with same parent as **L**).
→ If re-distribution fails, merge **L** and sibling.

If merge occurred, must delete entry (pointing to **L** or sibling) from parent of **L**.

Source: Chris Re

SELECTION CONDITIONS

The DBMS can use a B+Tree index if the query provides any of the attributes of the search key.

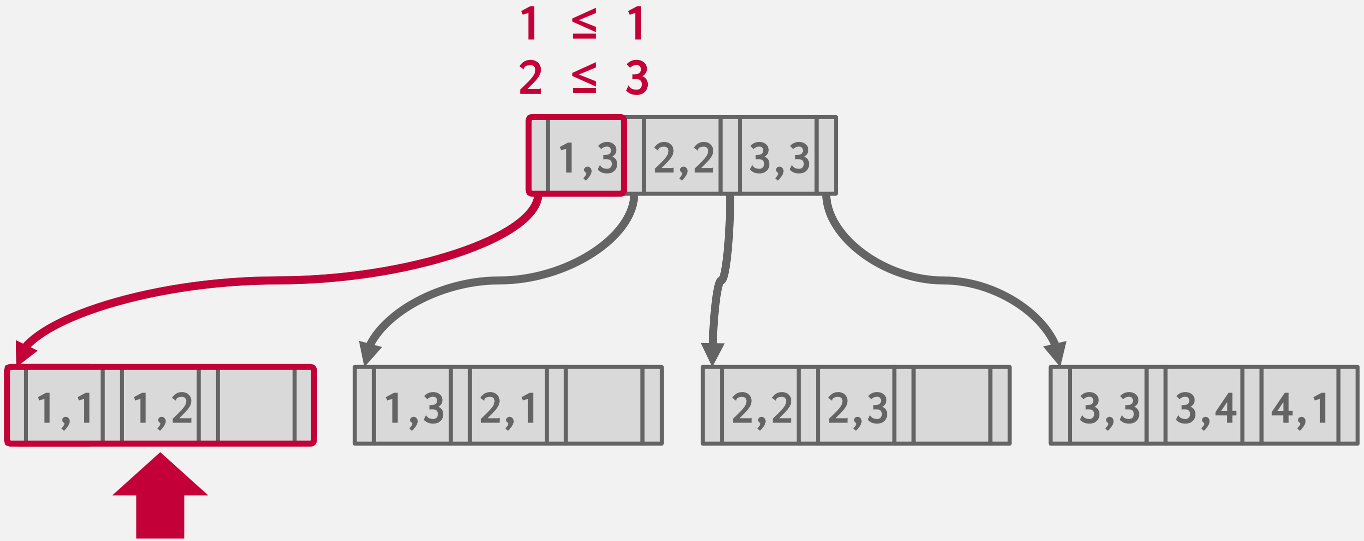
Example: Index on `<a,b,c>`
→ Supported `(a=1 AND b=2 AND c=3)`
→ Supported: `(a=1 AND b=2)`
→ Supported: `(b=2)`, `(c=3)`

Not all DBMSs support this.

For a hash index, we must have all attributes in search key.

SELECTION CONDITIONS

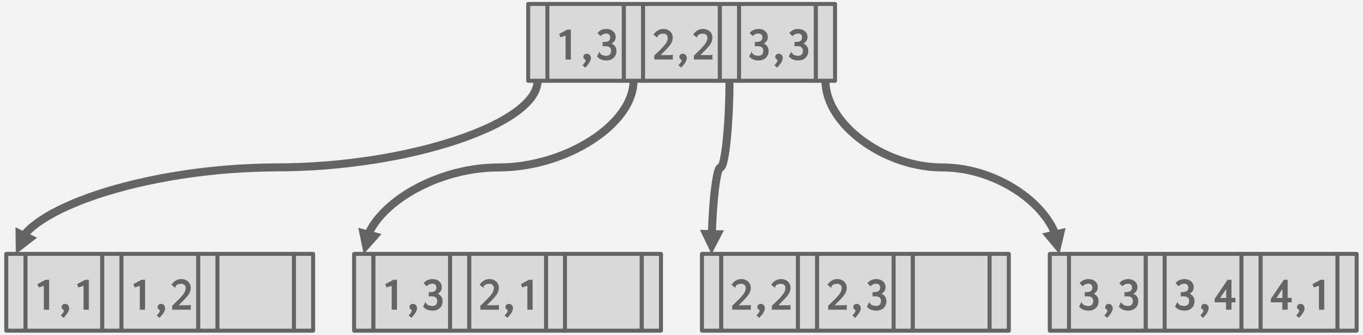
`Find Key=(1,2)`



SELECTION CONDITIONS

Find Key=(1,2)

Find Key=(1,*)



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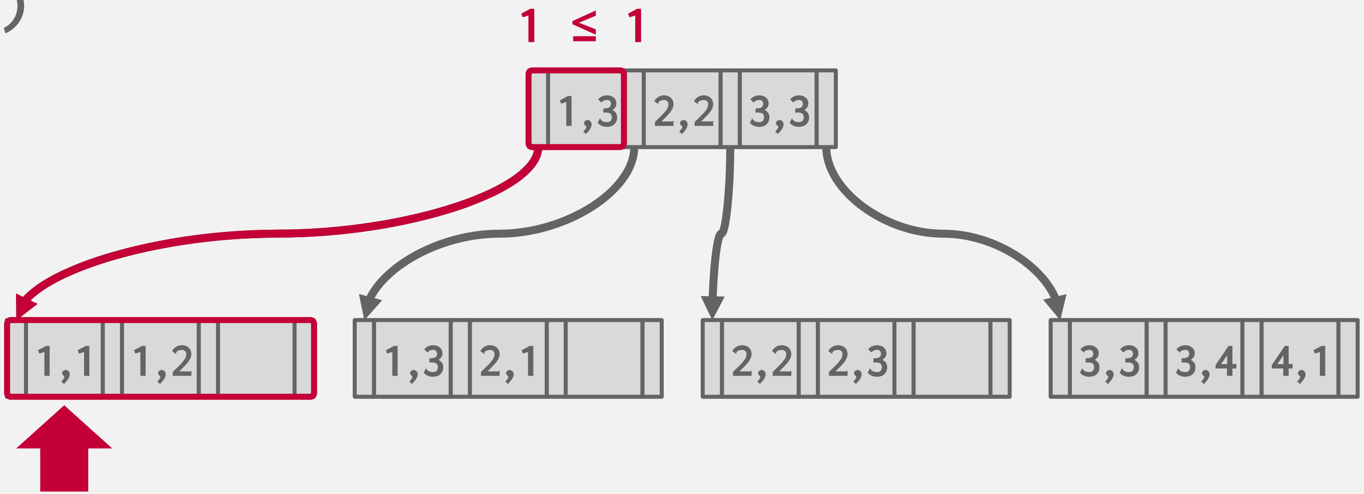

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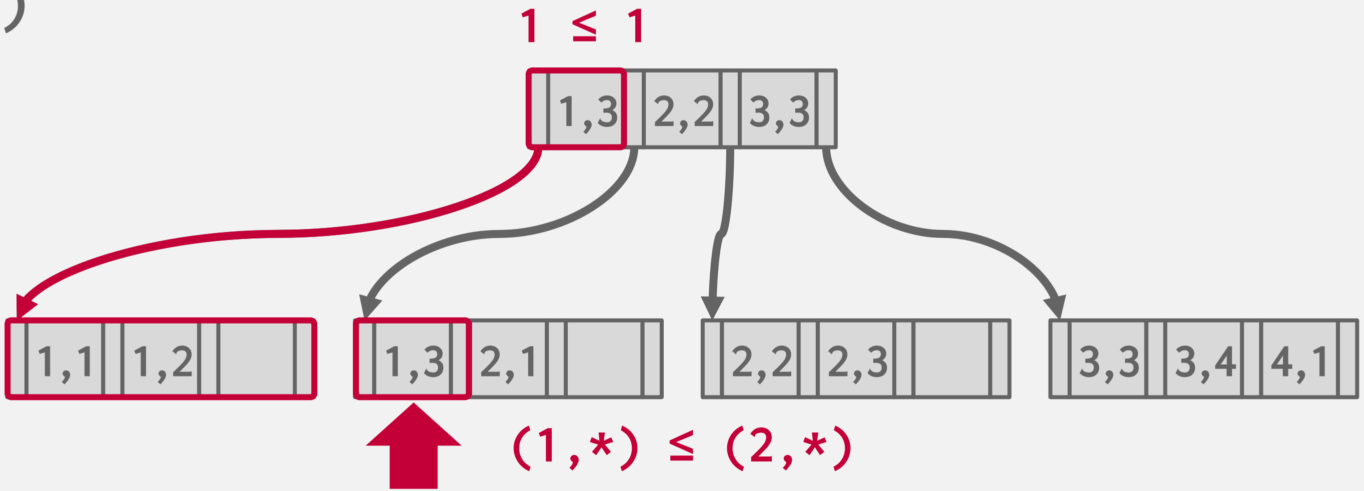

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```
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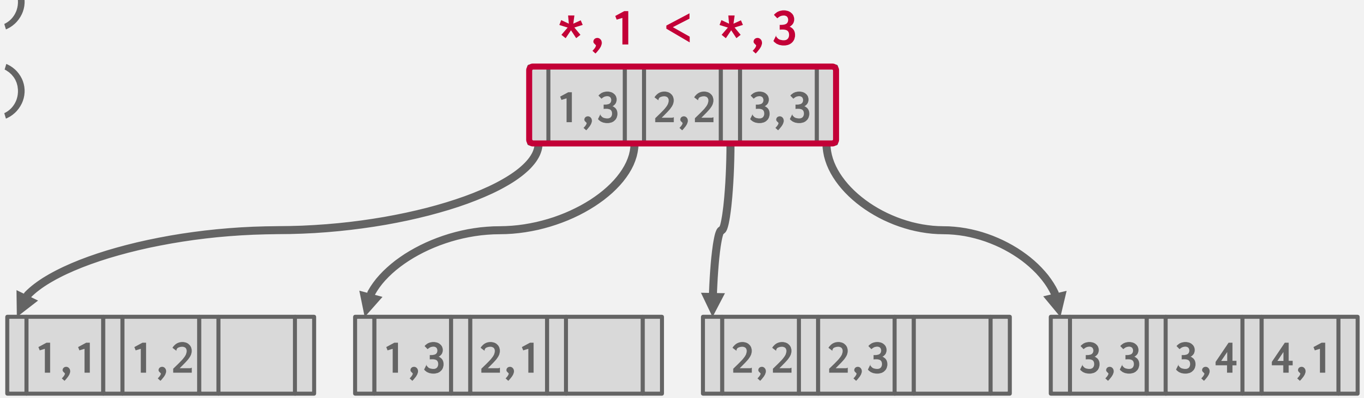

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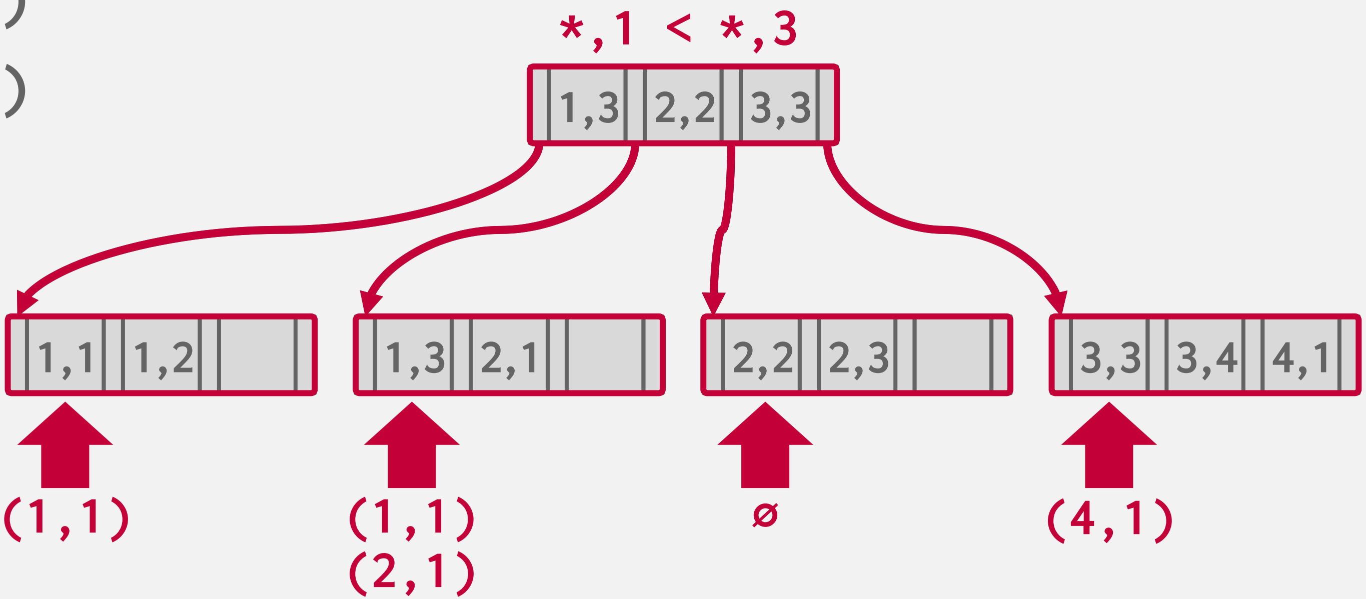


B+TREE – DUPLICATE KEYS

Approach #1: Append Record ID

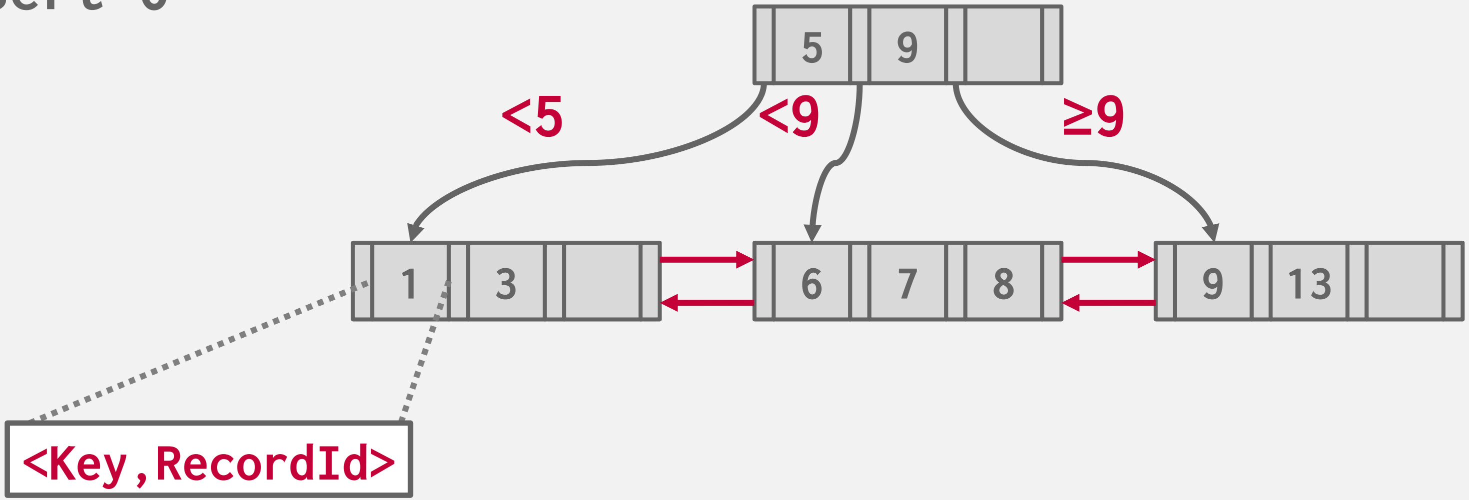
→ Add the tuple's unique Record ID as part of the key to ensure that all keys are unique.
→ The DBMS can still use partial keys to find tuples.

Approach #2: Overflow Leaf Nodes

→ Allow leaf nodes to spill into overflow nodes that contain the duplicate keys.
→ This is more complex to maintain and modify.

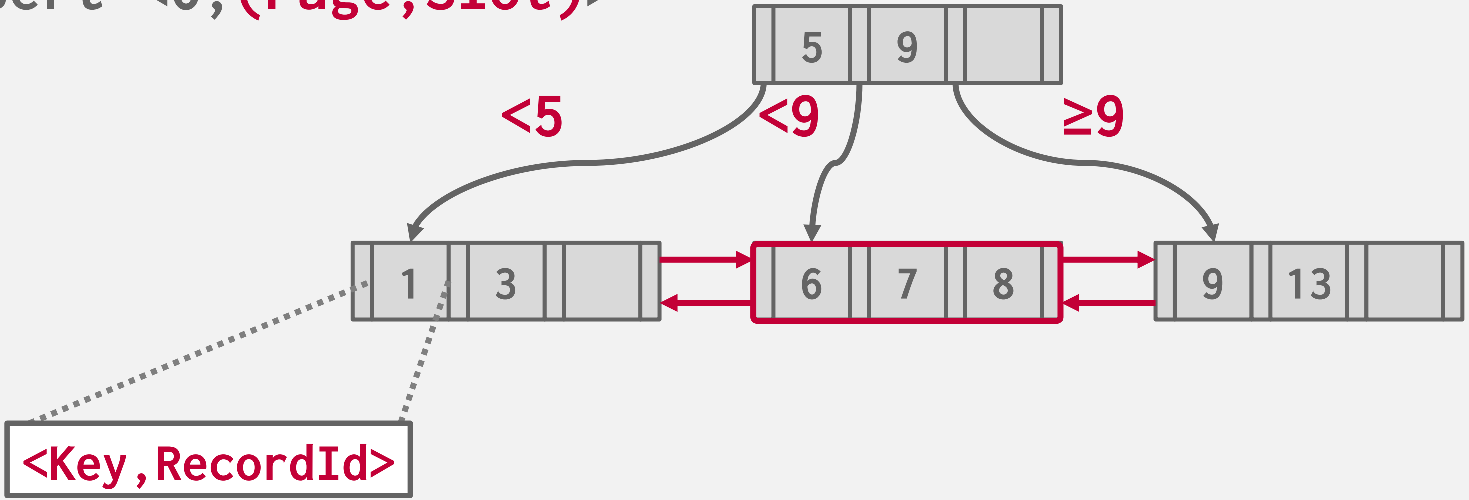
B+TREE – APPEND RECORD ID

Insert 6



B+TREE – APPEND RECORD ID

Insert <6,(Page,Slot)>



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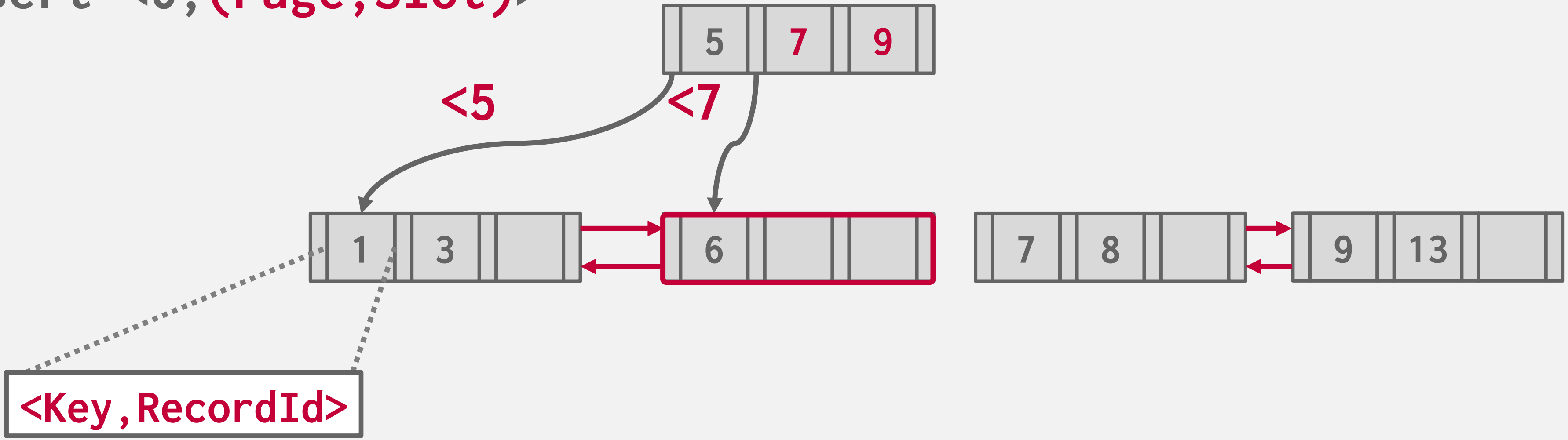

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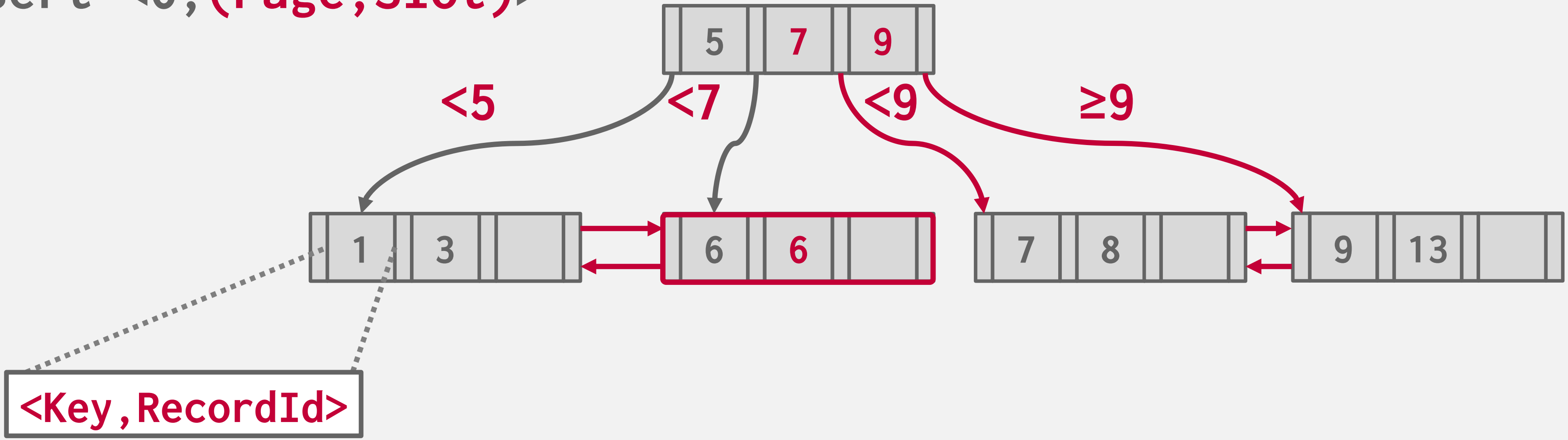
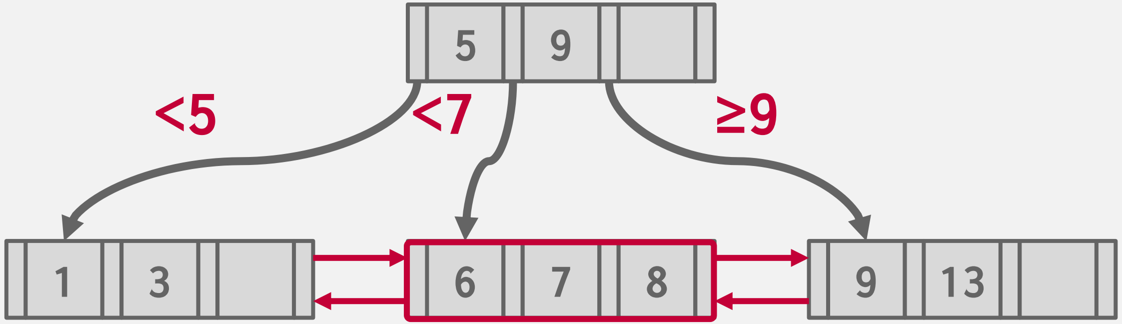


B+TREE – OVERFLOW LEAF NODES

Insert 6



B+TREE – OVERFLOW LEAF NODES

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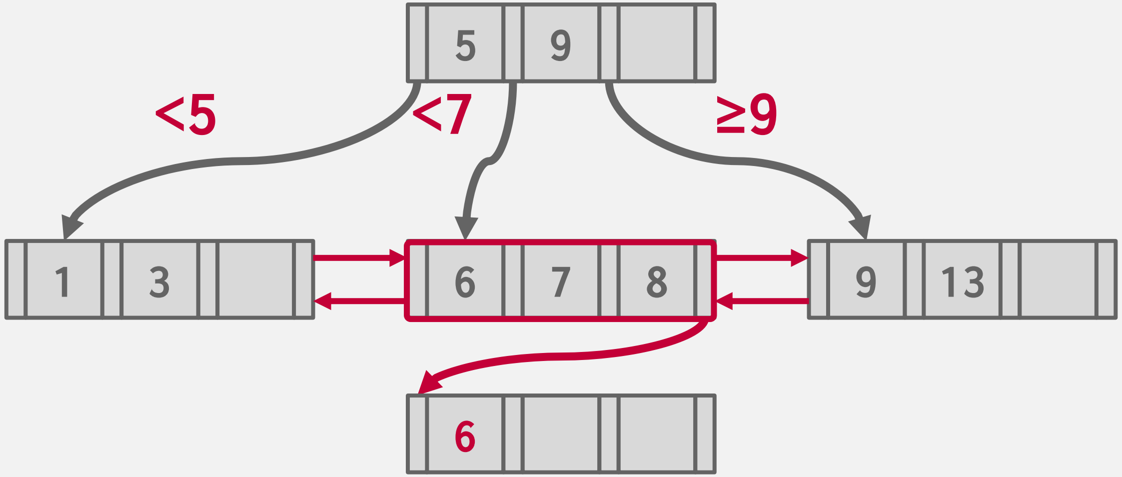
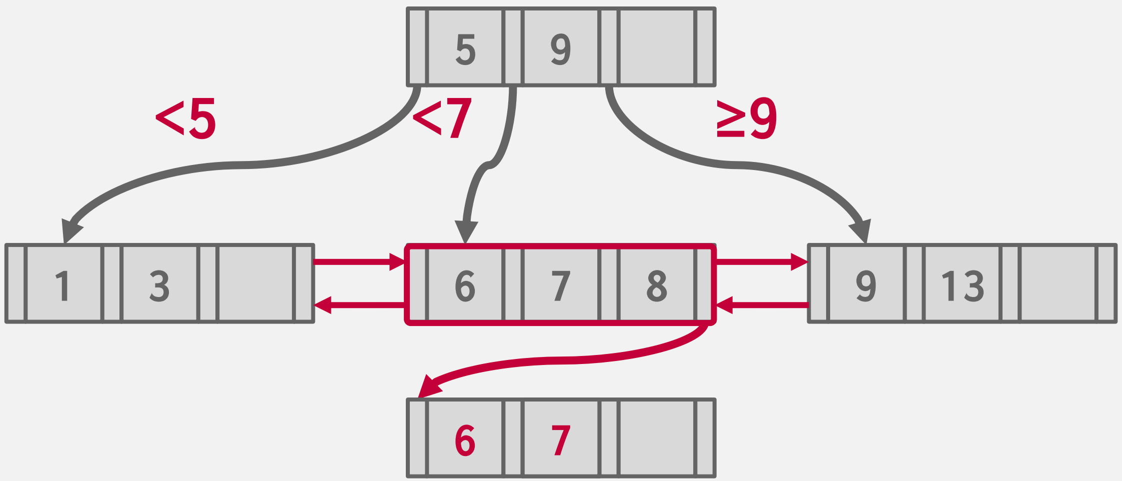


B+TREE – OVERFLOW LEAF NODES

Insert 6

Insert 7

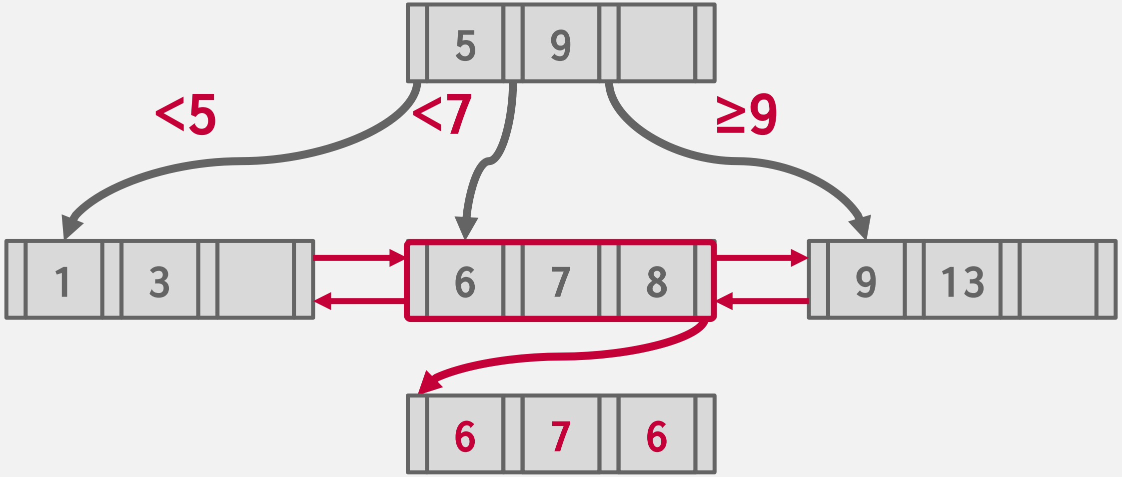


B+TREE – OVERFLOW LEAF NODES

Insert 6

Insert 7

Insert 6



CLUSTERED INDEXES

The table is stored in the sort order specified by the primary key.

→ Can be either heap- or index-organized storage.

Some DBMSs always use a clustered index.

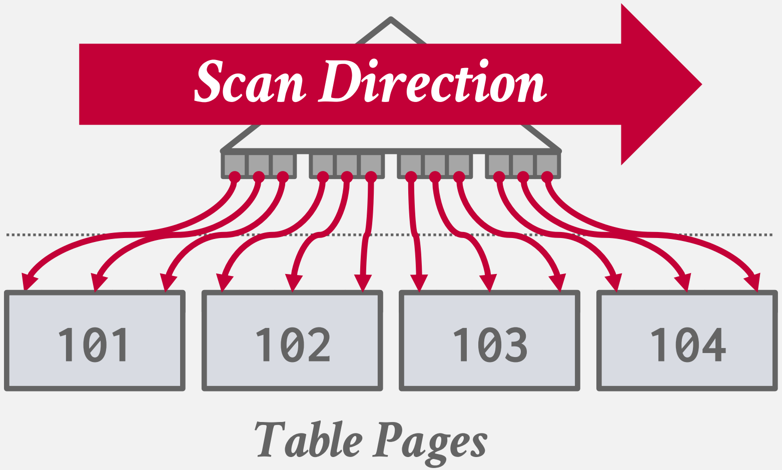
→ If a table does not contain a primary key, the DBMS will automatically make a hidden primary key.

Other DBMSs cannot use them at all.

CLUSTERED B+TREE

Traverse to the left-most leaf page and then retrieve tuples from all leaf pages.

This will always be better than sorting data for each query.

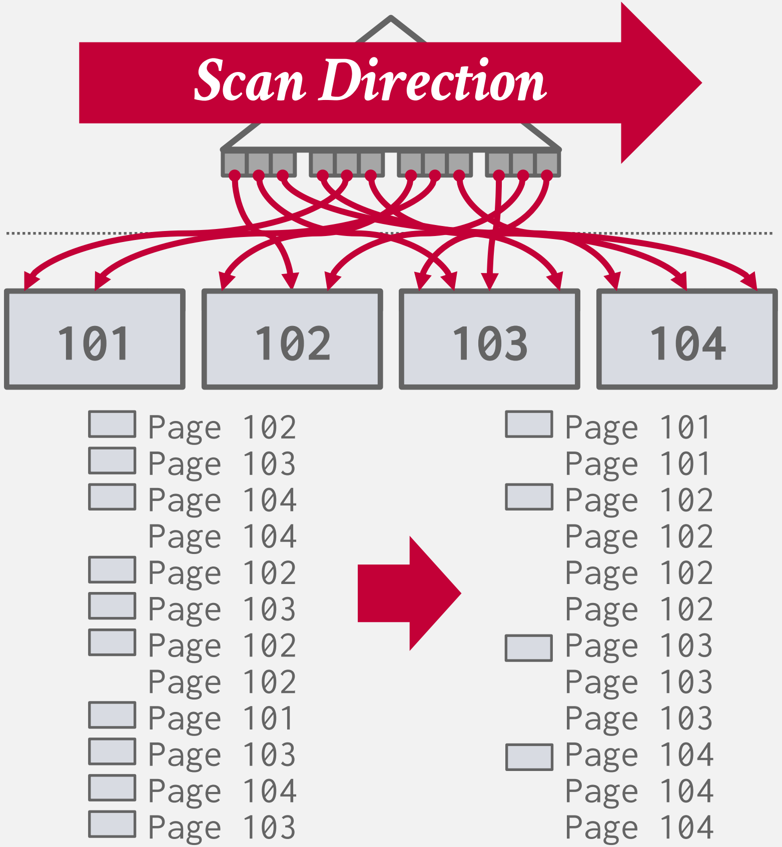


INDEX SCAN PAGE SORTING

Retrieving tuples in the order they appear in a non-clustered index is inefficient due to redundant reads.

A better approach is to find all the tuples that the query needs and then sort them based on their page ID.

The DBMS retrieves each page once.



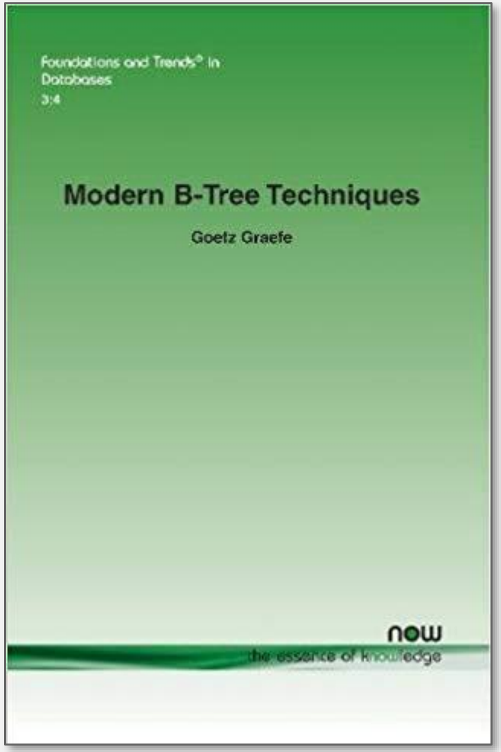
B+TREE DESIGN CHOICES

Node Size

Merge Threshold

Variable-Length Keys

Intra-Node Search



NODE SIZE

The slower the storage device, the larger the optimal node size for a B+Tree.

→ HDD: ~1MB
→ SSD: ~10KB
→ In-Memory: ~512B

Optimal sizes can vary depending on the workload

→ Leaf Node Scans vs. Root-to-Leaf Traversals

MERGE THRESHOLD

Some DBMSs do not always merge nodes when they are half full.

→ Average occupancy rate for B+Tree nodes is 69%.

Delaying a merge operation may reduce the amount of reorganization.

It may also be better to just let smaller nodes exist and then periodically rebuild entire tree.

This is why PostgreSQL calls their B+Tree a "non-balanced" B+Tree (nbtree).

VARIABLE-LENGTH KEYS

Approach #1: Pointers

→ Store the keys as pointers to the tuple's attribute.
→ Also called T-Trees (in-memory DBMSs)

Approach #2: Variable-Length Nodes

→ The size of each node in the index can vary.
→ Requires careful memory management.

Approach #3: Padding

→ Always pad the key to be max length of the key type.

Approach #4: Key Map / Indirection

→ Embed an array of pointers that map to the key + value list within the node.

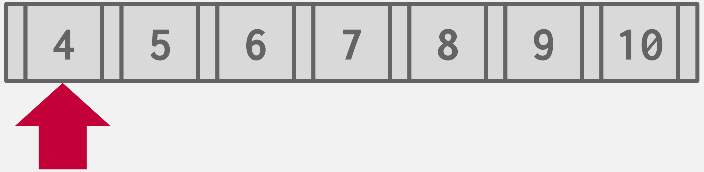
INTRA-NODE SEARCH

Approach #1: Linear

→ Scan node keys from beginning to end.

→ Use SIMD to vectorize comparisons.

Find Key=8



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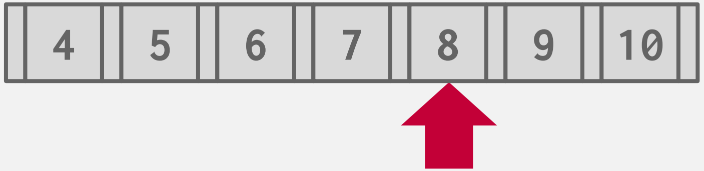

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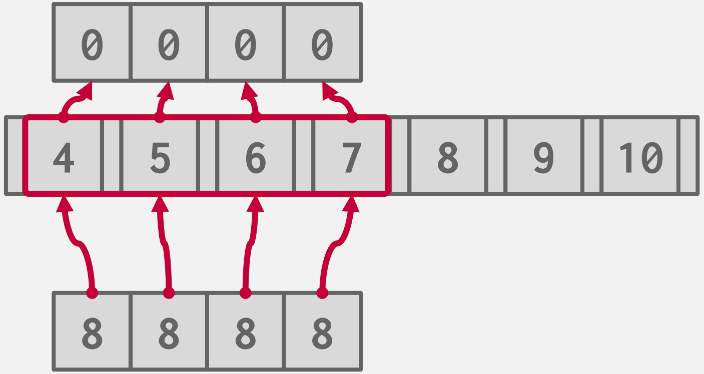

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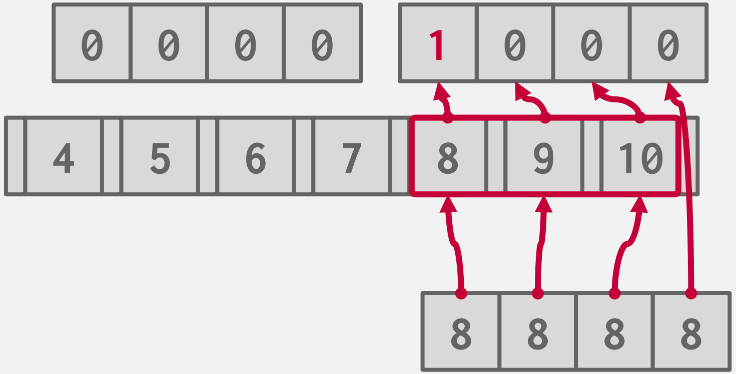


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INTRA-NODE SEARCH

Approach #1: Linear

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Approach #2: Binary

→ Jump to middle key, pivot left/right depending on comparison.

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Approach #2: Binary

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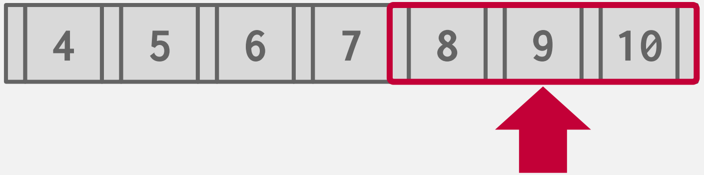


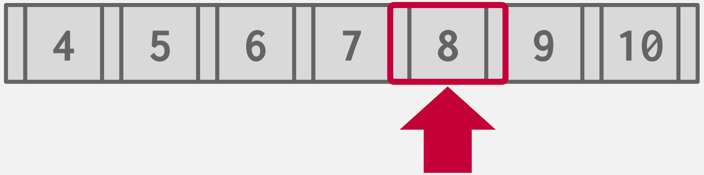
INTRA-NODE SEARCH

Approach #1: Linear

→ Scan node keys from beginning to end.
→ Use SIMD to vectorize comparisons.

Approach #2: Binary

→ Jump to middle key, pivot left/right depending on comparison.



INTRA-NODE SEARCH

Approach #1: Linear

→ Scan node keys from beginning to end.

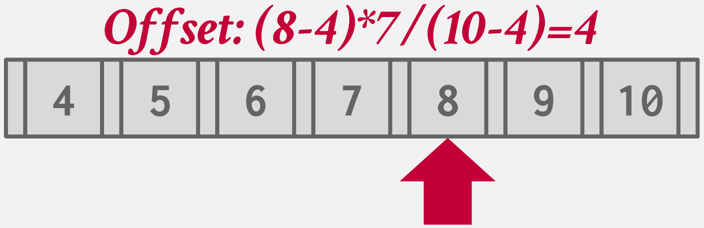
→ Use SIMD to vectorize comparisons.

Approach #2: Binary

→ Jump to middle key, pivot left/right depending on comparison.

Approach #3: Interpolation

→ Approximate location of desired key based on known distribution of keys.



OPTIMIZATIONS

Prefix Compression

Deduplication

Suffix Truncation

Pointer Swizzling

Bulk Insert

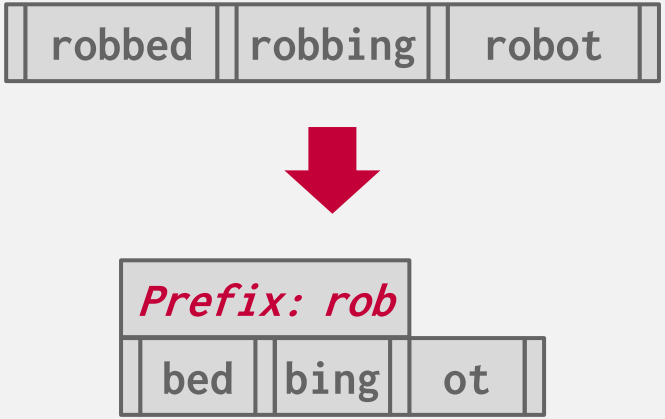
Buffered Updates

Many more…

PREFIX COMPRESSION

Sorted keys in the same leaf node are likely to have the same prefix.

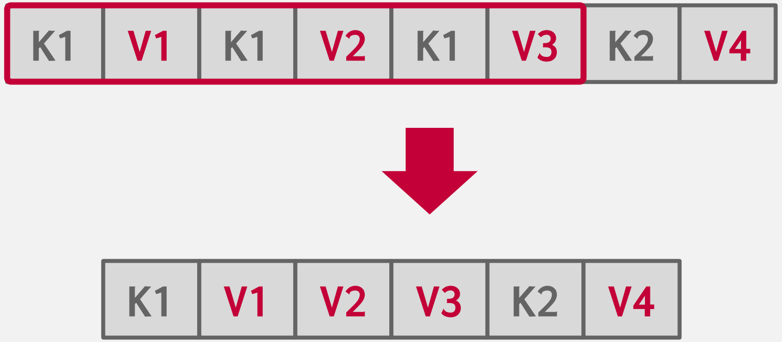
Instead of storing the entire key each time, extract common prefix and store only unique suffix for each key.
→ Many variations.



DEDUPLICATION

Non-unique indexes can end up storing multiple copies of the same key in leaf nodes.

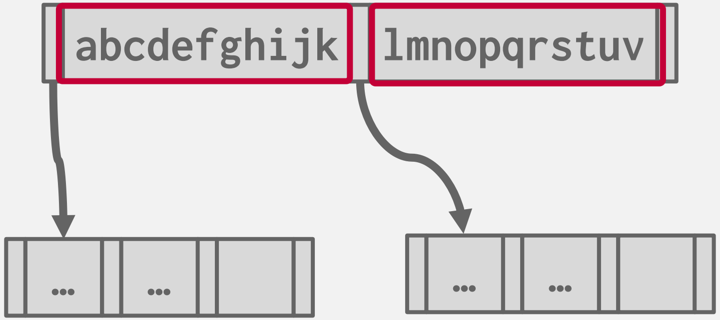
The leaf node can store the key once and then maintain a "posting list" of tuples with that key (similar to what we discussed for hash tables).



SUFFIX TRUNCATION

The keys in the inner nodes are only used to "direct traffic".
→ We don't need the entire key.

Store a minimum prefix that is needed to correctly route probes into the index.



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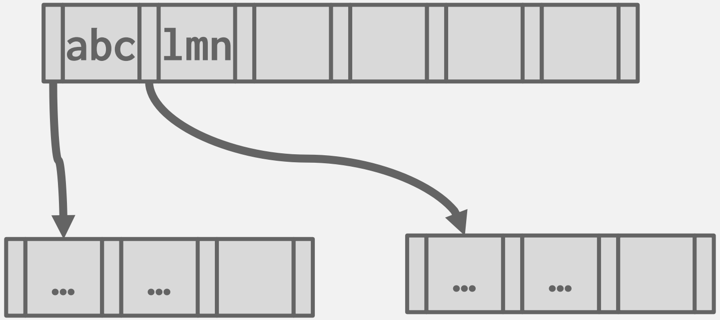
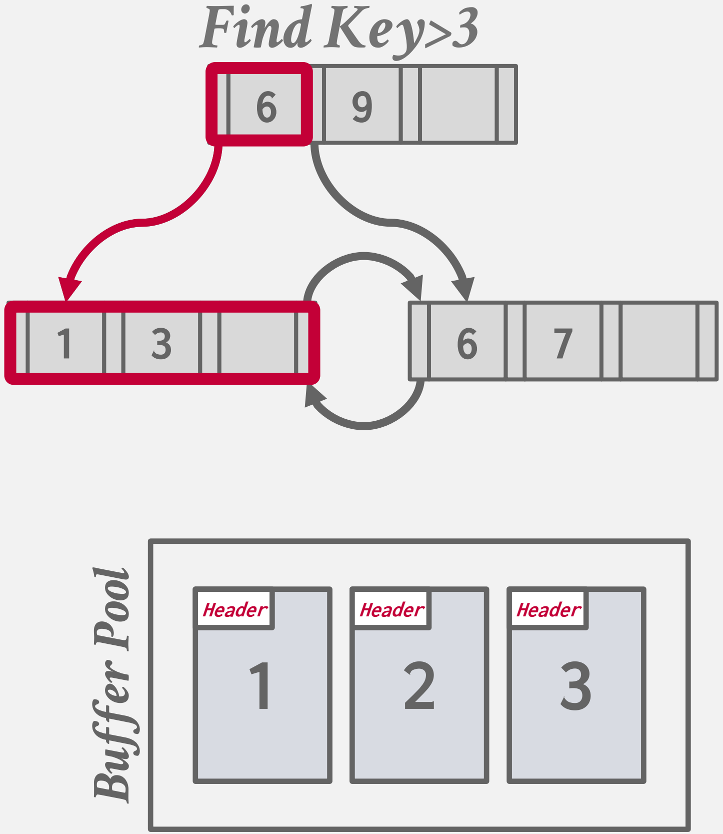


POINTER SWIZZLING

Nodes use page ids to reference other nodes in the index. The DBMS must get the memory location from the page table during traversal.

If a page is pinned in the buffer pool, then we can store raw pointers instead of page ids. This avoids address lookups from the page table.



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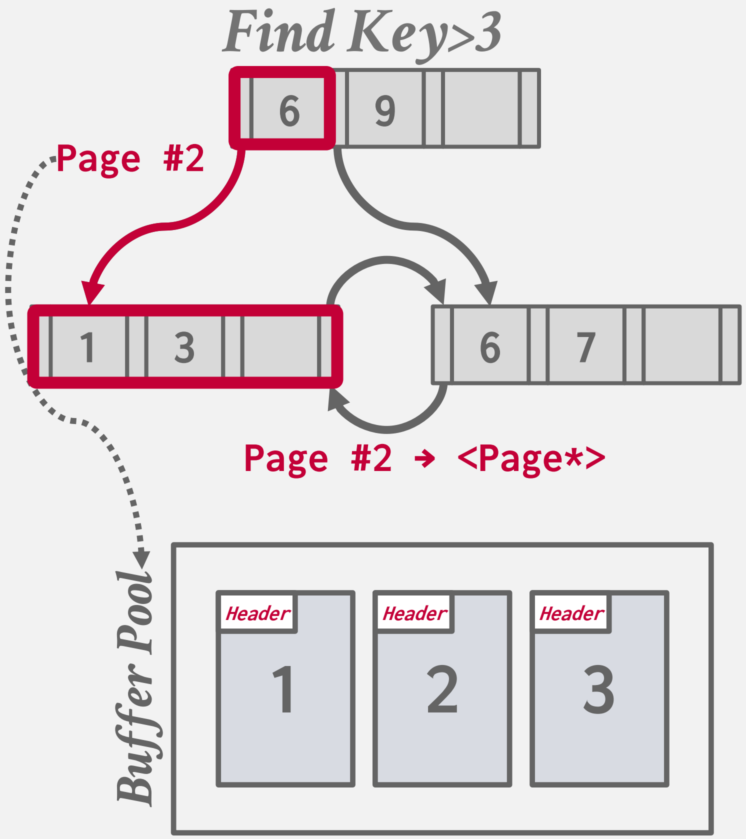

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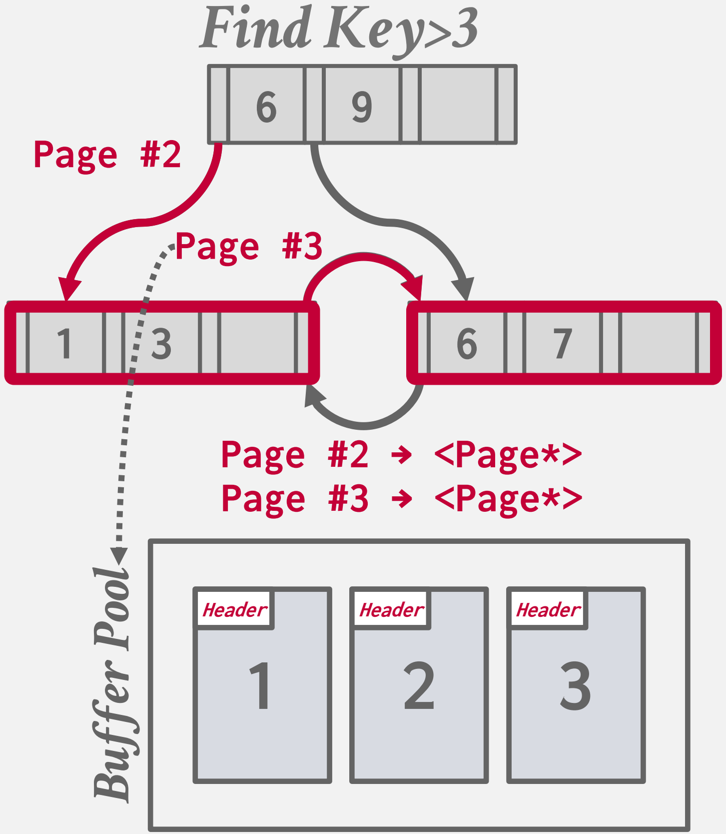

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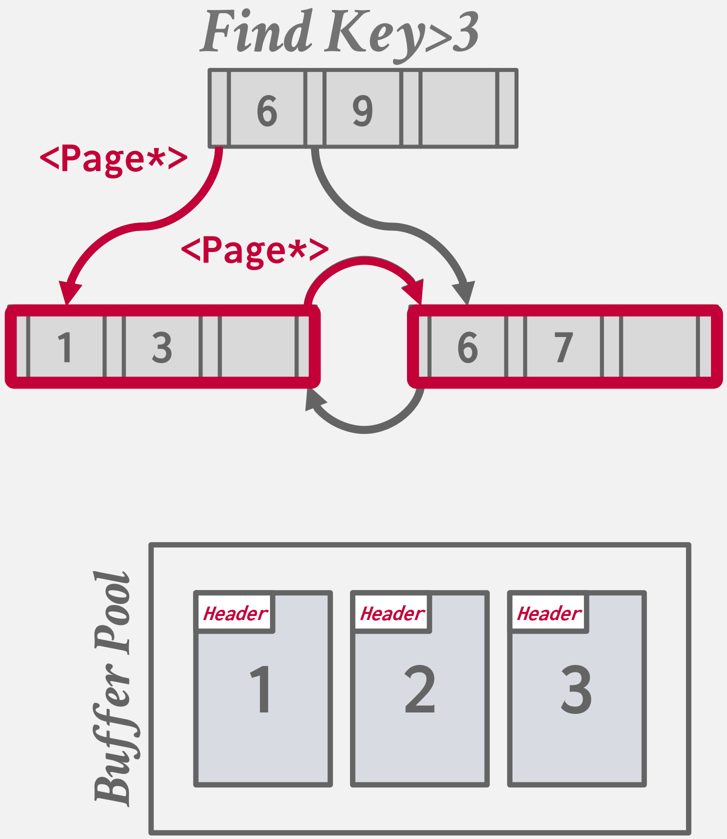
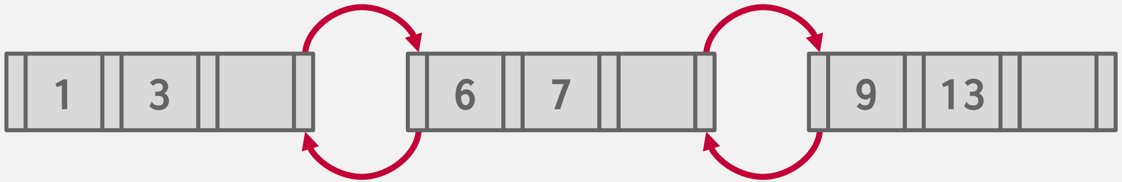


BULK INSERT

The fastest way to build a new B+Tree for an existing table is to first sort the keys and then build the index from the bottom up.

Keys: 3, 7, 9, 13, 6, 1

Sorted Keys: 1, 3, 6, 7, 9, 13



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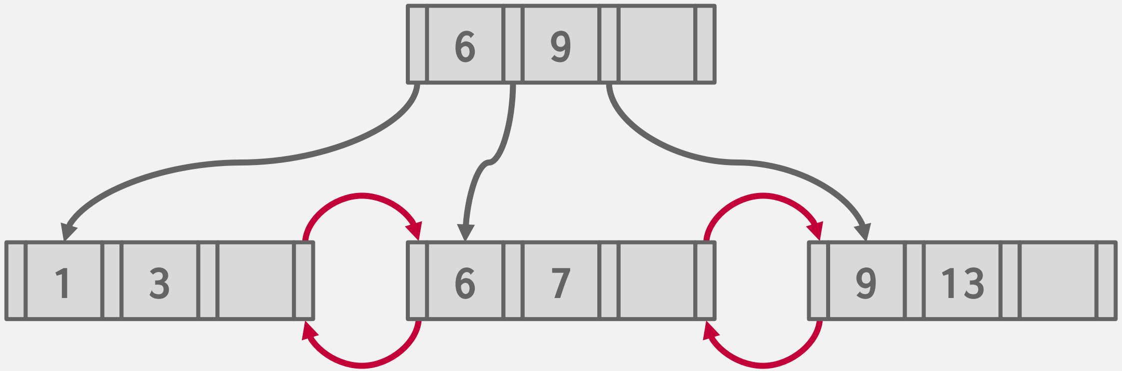


OBSERVATION

Modifying a B+tree is expensive when the DBMS has to split/merge nodes.

→ Worst case is when DBMS reorganizes the entire tree.
→ The worker that causes a split/merge is responsible for doing the work.

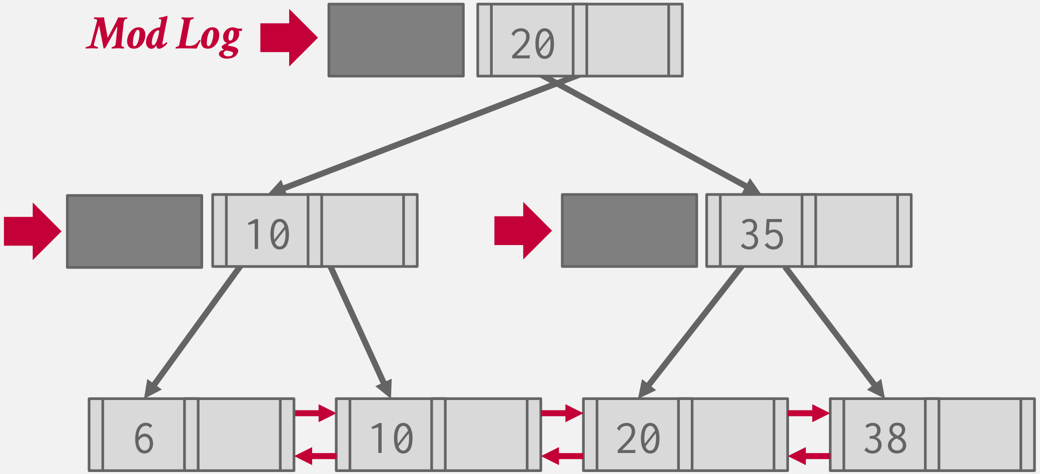
What if there was a way to delay updates and then apply multiple changes together in a batch?

WRITE-OPTIMIZED B+TREE

Instead of immediately applying updates, store changes to key/value entries in log buffers at inner nodes.

→ Also known as **Bε-trees**.

Updates cascade down to lower nodes incrementally when buffers get full.



Tokutek SPLINTERDB RelationalAI

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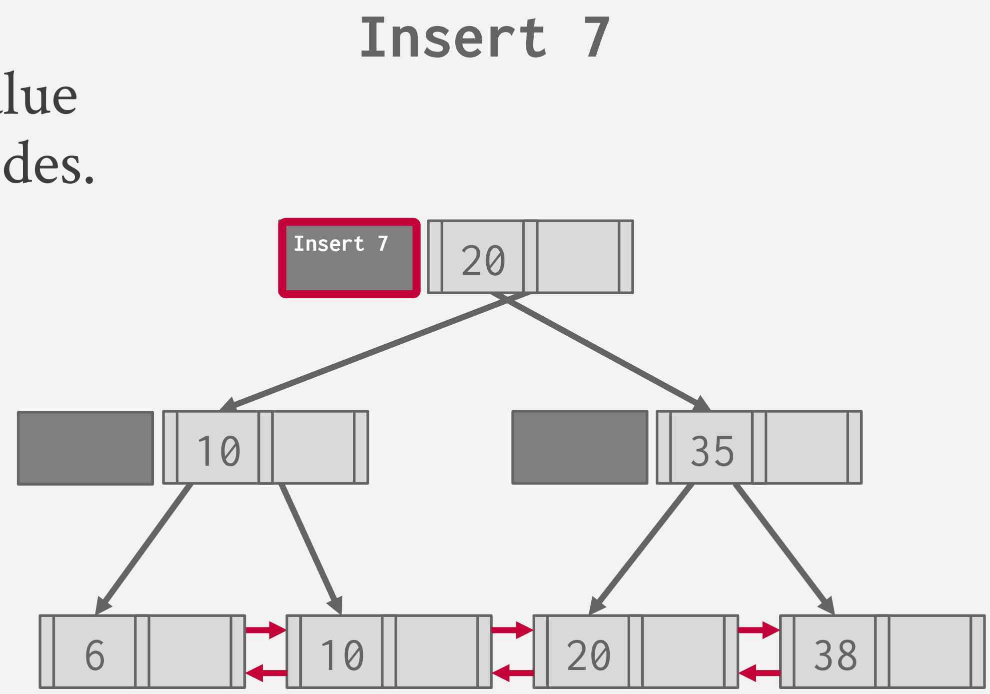

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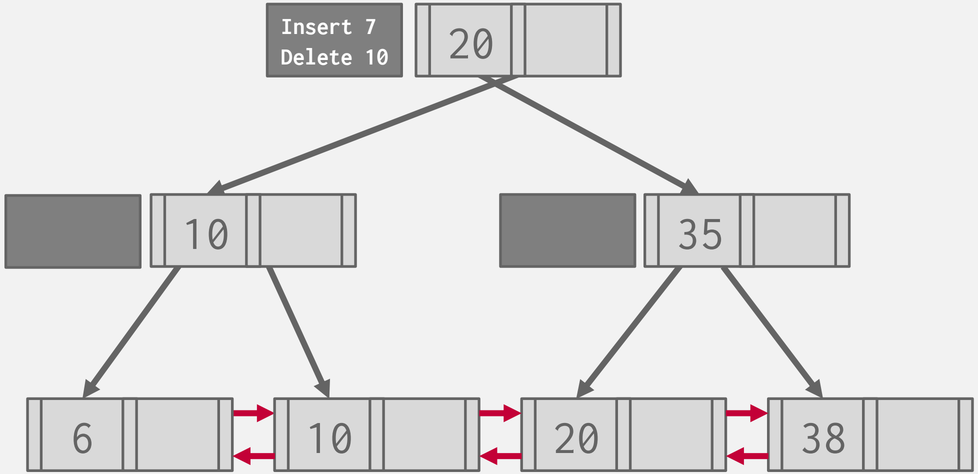
Insert 7
Delete 10



Tokutek SPLINTERDB RelationalAI

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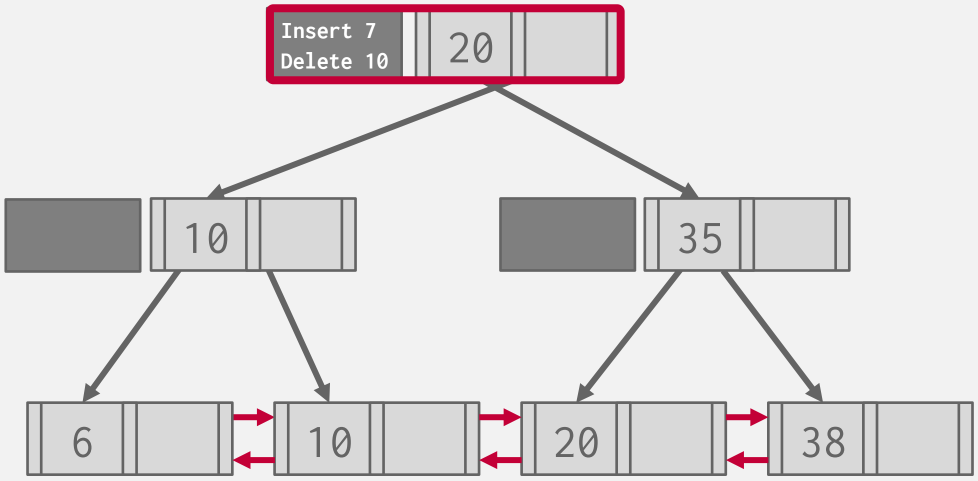
Find 10



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WRITE-OPTIMIZED B+TREE

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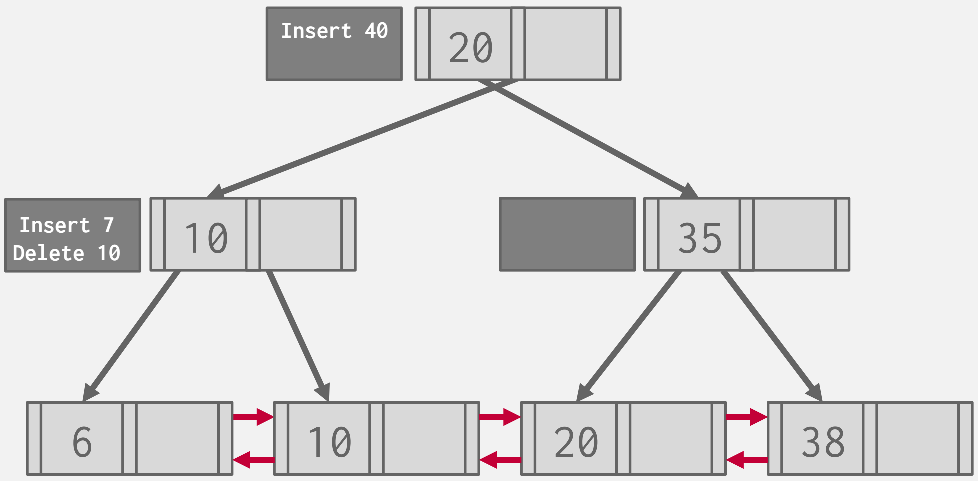
Insert 40



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DEMO

B+Tree vs. Hash Indexes

Table Clustering

CONCLUSION

The venerable B+Tree is (almost) always a good choice for your DBMS.

NEXT CLASS

Index Concurrency Control