Lecture #08

Sorting & Aggregations
UPCOMING EVENTS

Weaviate (ML↔DB Seminar)
→ Monday Oct 2\textsuperscript{nd} @ 4:30pm

FeatureForm (ML↔DB Seminar)
→ Monday Oct 9\textsuperscript{th} @ 4:30pm
**ADMINISTRIVIA**

- **Project #1** is due Sunday October 2\(^{nd}\) @ 11:59pm
  - Extra Office Hours: **Saturday October 1\(^{st}\) @ 3pm-5pm**

- **Homework #3** is due Sunday Oct 9th @ 11:59pm

- **Mid-Term Exam** is Wednesday Oct 13\(^{th}\)
  - During regular class time from 11:50-1:10pm
  - More details next week...
We are now going to talk about how to execute queries using the DBMS components we have discussed so far.

Next four lectures:
→ Operator Algorithms
→ Query Processing Models
→ Runtime Architectures

COURSE STATUS

Query Planning
Operator Execution
Access Methods
Buffer Pool Manager
Disk Manager
The operators are arranged in a tree.

Data flows from the leaves of the tree up towards the root.
→ We will discuss the granularity of the data movement next week.

The output of the root node is the result of the query.
DISK-ORIENTED DBMS

Just like it cannot assume that a table fits entirely in memory, a disk-oriented DBMS cannot assume that query results fit in memory.

We will use the buffer pool to implement algorithms that need to spill to disk.

We are also going to prefer algorithms that maximize the amount of sequential I/O.
WHY DO WE NEED TO SORT?

Relational model/SQL is unsorted.

Queries may request that tuples are sorted in a specific way (**ORDER BY**).

But even if a query does not specify an order, we may still want to sort to do other things:

→ Trivial to support duplicate elimination (**DISTINCT**).
→ Bulk loading sorted tuples into a B+Tree index is faster.
→ Aggregations (**GROUP BY**).
Most **database systems** use Quicksort for in-memory sorting.

In other **data platforms**, notably Python – the default sort algorithm is TimSort. It is a combination of insertion and binary merge sort. Often works well on real data.

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**IN-MEMORY SORTING**

If data fits in memory, then we can use a standard sorting algorithm like Quicksort.

If data does not fit in memory, then we need to use a technique that is aware of the cost of reading and writing disk pages...
https://www.toptal.com/developers/sorting-algorithms

https://visualgo.net/en/sorting
TODAY'S AGENDA

Top-N Heap Sort
External Merge Sort
Aggregations
TOP-N HEAP SORT

If a query contains an **ORDER BY** with a **LIMIT**, then the DBMS only needs to scan the data once to find the top-N elements.

Ideal scenario for **heapsort**: if the top-N elements fit in memory.
→ Scan data once, maintain an in-memory sorted priority queue.

```sql
SELECT * FROM enrolled
ORDER BY sid
FETCH FIRST 4 ROWS WITH TIES
```

**Original Data**

```
3 4 6 2 9 1 4 4 8
```

**Sorted Heap**

```
4 4 4 3 2 1
```

Output

Skip!

Skip and done!
EXTERNAL MERGE SORT

Divide-and-conquer algorithm that splits data into separate runs, sorts them individually, and then combines them into longer sorted runs.

Phase #1 – Sorting

→ Sort chunks of data that fit in memory and then write back the sorted chunks to a file on disk.

Phase #2 – Merging

→ Combine sorted runs into larger chunks.
A run is a list of key/value pairs.

**Key:** The attribute(s) to compare to compute the sort order.

**Value:** Two choices

→ Tuple (*early materialization*).
→ Record ID (*late materialization*).
We will start with a simple example of a 2-way external merge sort.

→ “2” is the number of runs that we are going to merge into a new run for each pass.

Data is broken up into $N$ pages.

The DBMS has a finite number of $B$ buffer pool pages to hold input and output data.
SIMPLIFIED 2-WAY EXTERNAL MERGE SORT

Pass #0
→ Read one page of the table into memory
→ Sort page into a “run” and write it back to disk
→ Repeat until the whole table has been sorted into runs

Pass #1,2,3,...
→ Recursively merge pairs of runs into runs twice as long
→ Need at least 3 buffer pages (2 for input, 1 for output)
SIMPLIFIED 2-WAY EXTERNAL MERGE SORT

In each pass, we read and write every page in the file.

Number of passes
= $1 + \lceil \log_2 N \rceil$

Total I/O cost
= $2N \cdot (# \text{ of passes)}$
SIMPLIFIED 2-WAY EXTERNAL MERGE SORT

This algorithm only requires three buffer pool pages to perform the sorting ($B=3$).
→ Two input pages, one output page

But even if we have more buffer space available ($B>3$), it does not effectively utilize them if the worker must block on disk I/O...
GENERAL EXTERNAL MERGE SORT

Pass #0
→ Use \( B \) buffer pages
→ Produce \( \left\lceil \frac{N}{B} \right\rceil \) sorted runs of size \( B \)

Pass #1,2,3,...
→ Merge \( B-1 \) runs (i.e., K-way merge)

Number of passes = \( 1 + \left\lfloor \log_{B-1} \left\lceil \frac{N}{B} \right\rceil \right\rfloor \)

Total I/O Cost = \( 2N \cdot (\# \text{ of passes}) \)
EXAMPLE

Determine how many passes it takes to sort 108 pages with 5 buffer pool pages: \( N=108, \ B=5 \)

\[ \rightarrow \text{Pass } \#0: \left\lceil \frac{N}{B} \right\rceil = \left\lceil \frac{108}{5} \right\rceil = 22 \text{ sorted runs of 5 pages each (last run is only 3 pages).} \]

\[ \rightarrow \text{Pass } \#1: \left\lceil \frac{N'}{B-1} \right\rceil = \left\lceil \frac{22}{4} \right\rceil = 6 \text{ sorted runs of 20 pages each (last run is only 8 pages).} \]

\[ \rightarrow \text{Pass } \#2: \left\lceil \frac{N''}{B-1} \right\rceil = \left\lceil \frac{6}{4} \right\rceil = 2 \text{ sorted runs, first one has 80 pages and second one has 28 pages.} \]

\[ \rightarrow \text{Pass } \#3: \text{Sorted file of 108 pages.} \]

\[ 1+\left\lceil \log_{B-1}\left\lceil \frac{N}{B} \right\rceil \right\rceil = 1+\left\lceil \log_4 22 \right\rceil = 1+\left\lceil 2.229... \right\rceil = 4 \text{ passes} \]
DOUBLE BUFFERING OPTIMIZATION

Prefetch the next run in the background and store it in a second buffer while the system is processing the current run.

→ Reduces the wait time for I/O requests at each step by continuously utilizing the disk.
DOUBLE BUFFERING

Prefetch the next run in the background and store it in a second buffer while the system is processing the current run.

→ Overlap CPU and I/O operations

Impact: reduces the effective “B” by half.

Reduces response time. What about throughput?
COMPARISON OPTIMIZATIONS

Approach #1: Code Specialization
→ Instead of providing a comparison function as a pointer to sorting algorithm, create a hardcoded version of sort that is specific to a key type.

Approach #2: Suffix Truncation
→ First compare a binary prefix of long VARCHAR keys instead of slower string comparison. Fallback to slower version if prefixes are equal.
USING B+TREES FOR SORTING

If the table that must be sorted already has a B+Tree index on the sort attribute(s), then we can use that to accelerate sorting.

Retrieve tuples in desired sort order by simply traversing the leaf pages of the tree.

Cases to consider:
→ Clustered B+Tree
→ Unclustered B+Tree
CASE #1 – CLUSTERED B+TREE

Traverse to the left-most leaf page, and then retrieve tuples from all leaf pages.

This is always better than external sorting because there is no computational cost, and all disk access is sequential.
CASE #2 – UNCLUSTERED B+TREE

Chase each pointer to the page that contains the data.

This is almost always a bad idea. In general, one I/O per data record.
AGGREGATIONS

Collapse values for a single attribute from multiple tuples into a single scalar value.

The DBMS needs a way to quickly find tuples with the same distinguishing attributes for grouping.

Two implementation choices:
→ Sorting
→ Hashing
### SELECT DISTINCT cid
FROM enrolled
WHERE grade IN ('B', 'C')
ORDER BY cid

```sql
SELECT DISTINCT cid 
FROM enrolled 
WHERE grade IN ('B', 'C') 
ORDER BY cid;
```

```sql
enrolled (sid, cid, grade)
```

<table>
<thead>
<tr>
<th>sid</th>
<th>cid</th>
<th>grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>53666</td>
<td>15-445</td>
<td>C</td>
</tr>
<tr>
<td>53688</td>
<td>15-721</td>
<td>A</td>
</tr>
<tr>
<td>53688</td>
<td>15-826</td>
<td>B</td>
</tr>
<tr>
<td>53666</td>
<td>15-721</td>
<td>C</td>
</tr>
<tr>
<td>53655</td>
<td>15-445</td>
<td>C</td>
</tr>
</tbody>
</table>

- **Filter**
- **Remove Columns**
- **Sort**
- **Eliminate Duplicates**

### Table

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
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<td>B</td>
</tr>
</tbody>
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</tr>
<tr>
<td>15-721</td>
</tr>
<tr>
<td>15-445</td>
</tr>
</tbody>
</table>

- **Sort**
ALTERNATIVES TO SORTING

What if we do not need the data to be ordered?
→ Forming groups in GROUP BY (no ordering)
→ Removing duplicates in DISTINCT (no ordering)

Hashing is a better alternative in this scenario.
→ Only need to remove duplicates, no need for ordering.
→ Can be computationally cheaper than sorting.
HASHING AGGREGATE

Populate an ephemeral hash table as the DBMS scans the table. For each record, check whether there is already an entry in the hash table:

→ **DISTINCT**: Discard duplicate

→ **GROUP BY**: Perform aggregate computation

If everything fits in memory, then this is easy.

If the DBMS must spill data to disk, then we need to be smarter...
EXTERNAL HASHING AGGREGATE

Phase #1 – Partition
→ Divide tuples into buckets based on hash key
→ Write them out to disk when they get full

Phase #2 – ReHash
→ Build in-memory hash table for each partition and compute the aggregation
PHASE #1 – PARTITION

Use a hash function $h_1$ to split tuples into partitions on disk.

→ A partition is one or more pages that contain the set of keys with the same hash value.
→ Partitions are “spilled” to disk via output buffers.

Assume that we have $B$ buffers.
We will use $B-1$ buffers for the partitions and 1 buffer for the input data.
PHASE #1 – PARTITION

SELECT DISTINCT cid
FROM enrolled
WHERE grade IN ('B', 'C')

enrolled (sid, cid, grade)

<table>
<thead>
<tr>
<th>sid</th>
<th>cid</th>
<th>grade</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
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<td>15-721</td>
<td>C</td>
</tr>
<tr>
<td>53655</td>
<td>15-445</td>
<td>C</td>
</tr>
</tbody>
</table>

Filter

Remove Columns

B-1 partitions

15-445
15-312

15-826
15-210

15-721
PHASE #2 – REHASH

For each partition on disk:
→ Read it into memory and build an in-memory hash table based on a second hash function $h_2$.
→ Then go through each bucket of this hash table to bring together matching tuples.

This assumes that each partition fits in memory.
PHASE #2 – REHASH

SELECT DISTINCT cid
FROM enrolled
WHERE grade IN ('B', 'C')

<table>
<thead>
<tr>
<th>sid</th>
<th>cid</th>
<th>grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>53666</td>
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<td>15-721</td>
<td>C</td>
</tr>
<tr>
<td>53655</td>
<td>15-445</td>
<td>C</td>
</tr>
</tbody>
</table>

Phase #1 Buckets

B-1 Partitions

15-445 15-445
15-445 15-445
15-445 15-445
15-826 15-826
15-721

Hash Table

<table>
<thead>
<tr>
<th>cid</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-445</td>
</tr>
<tr>
<td>15-826</td>
</tr>
<tr>
<td>15-721</td>
</tr>
</tbody>
</table>

Final Result

<table>
<thead>
<tr>
<th>cid</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-445</td>
</tr>
<tr>
<td>15-826</td>
</tr>
<tr>
<td>15-721</td>
</tr>
</tbody>
</table>
HASHING SUMMARIZATION

During the rehash phase, store pairs of the form (GroupKey → RunningVal)

When we want to insert a new tuple into the hash table:
→ If we find a matching GroupKey, just update the RunningVal appropriately
→ Else insert a new GroupKey → RunningVal
HASHING SUMMARIZATION

```
SELECT cid, AVG(s.gpa) 
FROM student AS s, enrolled AS e 
WHERE s.sid = e.sid 
GROUP BY cid
```

**Phase #1**

Buckets

| 15-445 | h2 |
| 15-445 |
| 15-826 | h2 |
| : |
| 15-721 | h2 |

**Hash Table**

<table>
<thead>
<tr>
<th>key</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-445</td>
<td>(2, 7.32)</td>
</tr>
<tr>
<td>15-826</td>
<td>(1, 3.33)</td>
</tr>
<tr>
<td>15-721</td>
<td>(1, 2.89)</td>
</tr>
</tbody>
</table>

**Final Result**

<table>
<thead>
<tr>
<th>cid</th>
<th>AVG(gpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-445</td>
<td>3.66</td>
</tr>
<tr>
<td>15-826</td>
<td>3.33</td>
</tr>
<tr>
<td>15-721</td>
<td>2.89</td>
</tr>
</tbody>
</table>

**Running Totals**

<table>
<thead>
<tr>
<th>Function</th>
<th>Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>AVG(col)</td>
<td>(COUNT, SUM)</td>
</tr>
<tr>
<td>MIN(col)</td>
<td>(MIN)</td>
</tr>
<tr>
<td>MAX(col)</td>
<td>(MAX)</td>
</tr>
<tr>
<td>SUM(col)</td>
<td>(SUM)</td>
</tr>
<tr>
<td>COUNT(col)</td>
<td>(COUNT)</td>
</tr>
</tbody>
</table>

"Running Totals"
CONCLUSION

Choice of sorting vs. hashing is subtle and depends on optimizations done in each case.

We already discussed the optimizations for sorting:
→ Chunk I/O into large blocks to amortize costs
→ Double-buffering to overlap CPU and I/O
NEXT CLASS

Nested Loop Join
Sort-Merge Join
Hash Join