Carnegie Mellon University

# Database Systems

Bloom Filters, Tries, Skip Lists, Inverted Indexes, Vector Indexes



# **ADMINISTRIVIA**

- **Project #1** is due Sunday Sept 29<sup>th</sup> @ 11:59pm
- → Recitation + Profiling Tutorial: <u>@160</u>
- → Extra Office Hours on Saturday Sept 28<sup>th</sup> @ 3:00-5:00pm
- → Location: GHC 5207
- → <a href="https://ohq.eberly.cmu.edu">https://ohq.eberly.cmu.edu</a>
- **Homework #3** is due Sunday Sept 6<sup>th</sup> @ 11:59pm
- Mid-term Exam on Wednesday Oct 9th @ 2:00pm
- $\rightarrow$  In-class in this room.
- $\rightarrow$  More info next week.



# INDEXES VS. FILTERS

An **index** data structure of a subset of a table's attributes that are organized and/or sorted to the location of specific tuples using those attributes.

 $\rightarrow$  Example: B+Tree

A **filter** is a data structure that answers set membership queries; it tells you whether a key (likely) exists in a set but <u>not</u> where it is located.

→ Example: Bloom Filter



# TODAY'S AGENDA

Bloom Filters

Skip Lists

Tries / Radix Trees

Inverted Indexes

Vector Indexes

DB Flash Talk: TiDB



Probabilistic data structure (bitmap) that answers set membership queries.

- $\rightarrow$  False negatives will never occur.
- $\rightarrow$  False positives can sometimes occur.
- $\rightarrow$  See Bloom Filter Calculator.

### Insert(x):

 $\rightarrow$  Use *k* hash functions to set bits in the filter to 1.

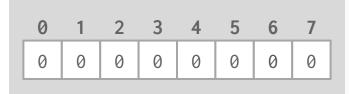
### Lookup(x):

 $\rightarrow$  Check whether the bits are 1 for each hash function.



#### Insert 'RZA'

#### Bloom Filter

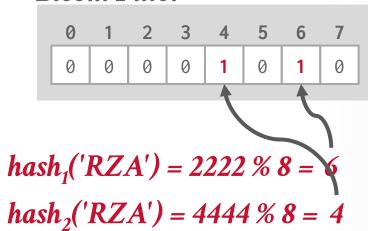


$$hash_1(RZA') = 2222 \% 8 = 6$$

$$hash_2('RZA') = 4444 \% 8 = 4$$

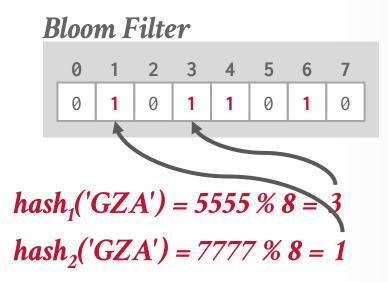
#### Insert 'RZA'

# Bloom Filter



Insert 'RZA'

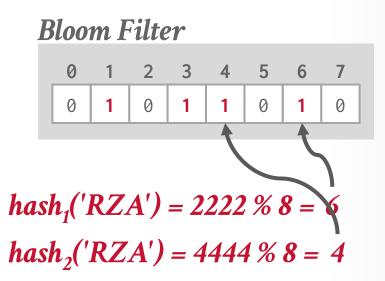
Insert 'GZA'



Insert 'RZA'

Insert 'GZA'

Lookup 'RZA' → *TRUE* 



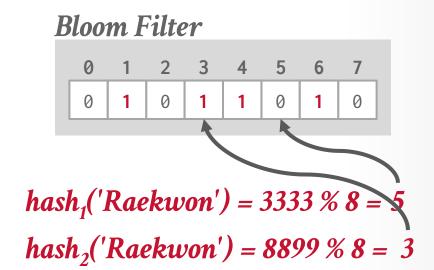


Insert 'RZA'

Insert 'GZA'

Lookup 'RZA' → TRUE

Lookup 'Raekwon'→ *FALSE* 





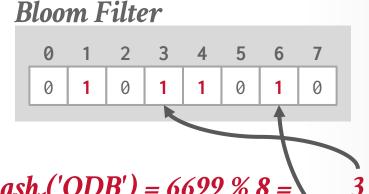
Insert 'RZA'

Insert 'GZA'

Lookup 'RZA' → TRUE

Lookup 'Raekwon'→ *FALSE* 

Lookup 'ODB' → *TRUE* 





# OTHER FILTERS

### **Counting Bloom Filter**

- → Supports dynamically adding and removing keys.
- → Uses integers instead of bits to count the number of occurrences of a key in a set.



### **Cuckoo Filter**

- → Also supports dynamically adding and removing keys.
- → Uses a Cuckoo Hash Table but stores <u>fingerprints</u> instead of full keys.



### Succinct Range Filter (SuRF)

→ Immutable compact trie that supports approximate exact matches and range filtering.



# **OTHER**

### **Counting Bloom Filter**

- → Supports dynamically adding
- $\rightarrow$  Uses integers instead of bits t occurrences of a key in a set.

# Carnegie Mellon University

### **Cuckoo Filter**

- $\rightarrow$  Also supports dynamically a
- → Uses a Cuckoo Hash Table b of full keys.



# Succinct Range Filter (S

→ Immutable compact trie tha matches and range filtering

#### Develop with Redis Quick starts Connect Understand data types Strings Lists Sets Hashes Sorted sets Streams Geospatial Bitmaps Bitfields Probabilistic HyperLogLog Bloom filter Cuckoo filter t-digest Top-K Count-min sketch Configuration Time series Interact with data Libraries and tools Redis products Commands

Docs → Develop with Redis → Understand Redis data types → Probabilistic → Cuckoo filter

#### Cuckoo filter

Cuckoo filters are a probabilistic data structure that checks for presence of an element in a set

A Cuckoo filter, just like a Bloom filter, is a probabilistic data structure in Redis Stack that enables you to check if an element is present in a set in a very fast and space efficient way, while also allowing for deletions and showing better performance than Bloom in some

While the Bloom filter is a bit array with flipped bits at positions decided by the hash function, a Cuckoo filter is an array of buckets, storing fingerprints of the values in one of the buckets at positions decided by the two hash functions. A membership query for item x searches the possible buckets for the fingerprint of x, and returns true if an identical fingerprint is found. A cuckoo filter's fingerprint size will directly determine the false positive rate.

Use cases

#### Targeted ad campaigns (advertising, retail)

This application answers this question: Has the user signed up for this campaign yet?

Use a Cuckoo filter for every campaign, populated with targeted users' ids. On every visit, the user id is checked against one of the Cuckoo filters,

- · If yes, the user has not signed up for campaign. Show the ad.
- If the user clicks ad and signs up, remove the user id from that Cuckoo filter.
- If no, the user has signed up for that campaign. Try the next ad/Cuckoo filter.

Discount code/coupon validation (retail, online shops)

This application answers this question: Has this discount code/coupon been used yet?

Use a Cuckoo filter populated with all discount codes/coupons. On every try, the entered code

- · If no, the coupon is not valid.
- If yes, the coupon can be valid. Check the main database. If valid, remove from Cuckoo

#### **SECMU·DB**

# **OBSERVATION**

The easiest way to implement a dynamic orderpreserving index is to use a sorted linked list.

All operations have to linear search.

 $\rightarrow$  Average Cost: O(n)



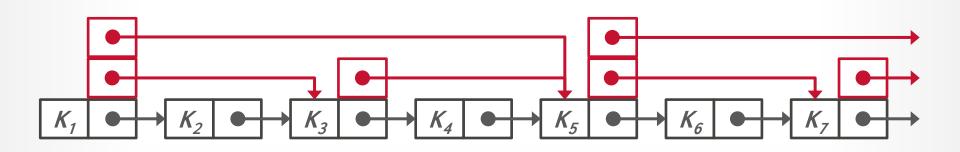


# **OBSERVATION**

The easiest way to implement a dynamic orderpreserving index is to use a sorted linked list.

All operations have to linear search.

 $\rightarrow$  Average Cost: O(n)





# SKIP LISTS

Multiple levels of linked lists with extra pointers to skip over entries.

- $\rightarrow$  1<sup>st</sup> level is a sorted list of all keys.
- $\rightarrow$  2<sup>nd</sup> level links every other key
- $\rightarrow$  3<sup>rd</sup> level links every fourth key
- $\rightarrow$  Each level has  $\frac{1}{2}$  the keys of one below it

Maintains keys in sorted order without requiring global rebalancing.

 $\rightarrow$  Approximate O(log n) search times.

Mostly for in-memory data structures.

→ Example: LSM MemTable

#### Skip Lists: A Probabilistic Alternative to Balanced Trees

Skip lists are a data structure that can be used in place of balanced trees. Skip lists use probabilistic balancing rather than strictly enforced balancing and as a result the algorithms for bisertion and deletion in skip lists are much simpler and significantly faster than equivalent algorithms for balanced trees.

#### William Pugh

Binary trees can be used for representing abstant data types such as discinnates and ordered bins. Divy work well when the elements are inserted in a random order. Some sequences of operations, such as inserting the elements in order, produce degenerate data structures that give very proor performance. If it was proudile to standarding parameta that for a form as to import the parameters of the contract of the

mance. Sale fars are a probabilistic alternative to balanced trees. Skip its as are balanced by consulting a random number garacter and the sale of t

to be random.

Balancing a data structure probabilistically is exsist than explicitly maintaining the balanci. For many applications, explicitly maintaining the balance. For many applications, and the structure of the structure

#### SKIPLIST

We might need to examine every node of the list when searching a linked list (Figuer 1a). If the list is stored in sorted order and every other node of the list also has a pointer to the node two abead it in the list (Figuer 1b), we have to examine no more than (Rightarrow 1a). We have to examine no Also giving every fourth node a pointer four ahead (Figure 1c) requires that no more than |n4| + 2 nodes be examined. If every (23)<sup>6</sup> nodes has a pointer 2 nodes shead Figure 1d), the number of nodes that must be examined can be reduced to |n6| + 100 nodes. The figure 1d), the number of pointers. This data structure could be used for fast searching, but insertion and deletion would be impracticed.

and decided with the imprecision with called a level is noted. The tree of  $(1/2)^2$  mode that a pointer  $(2)^2$  mode and the level of solor and sharehold in a simple parter.  $(3/2)^2$  and level  $(1/2)^2$  mode  $(3/2)^2$  mode (3/2

#### SKIP LIST ALGORITHMS

This section gives algorithms to search for, insert and delete elements in a distributory or symbol table. The Survach operation neutrum the contents of the value associated with the desired key or judipare of the key is not persent. The Interest tion succinities a specified key with a new value (inverting for the personal personal personal personal personal personal perdefents the specified key. It is may to support additional operations when the personal personal personal personal personal and the personal perso

is closer a madeomity when the node is inserted without regard for the number of elements in the data structure. A ever'd node has fewer pleasters, advected 1 throught. We do not need to store the level of a node in the node. Levels as copped at some appropriate constant ManZevel. The level of a list is the maximum level currently in the list conjunction of the comply. The loader of a list has forward pointers at levels on through ManZevel. The fewer and pointers of the beader at levels higher than the current maximum level of the list point levels higher than the current maximum level of the list point.

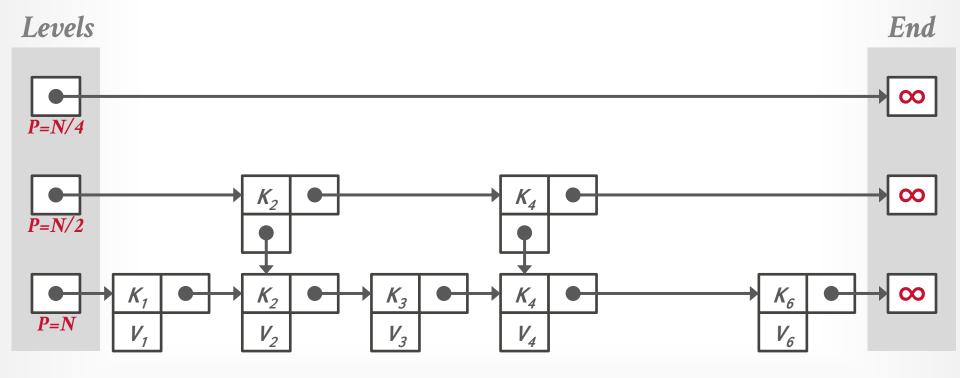




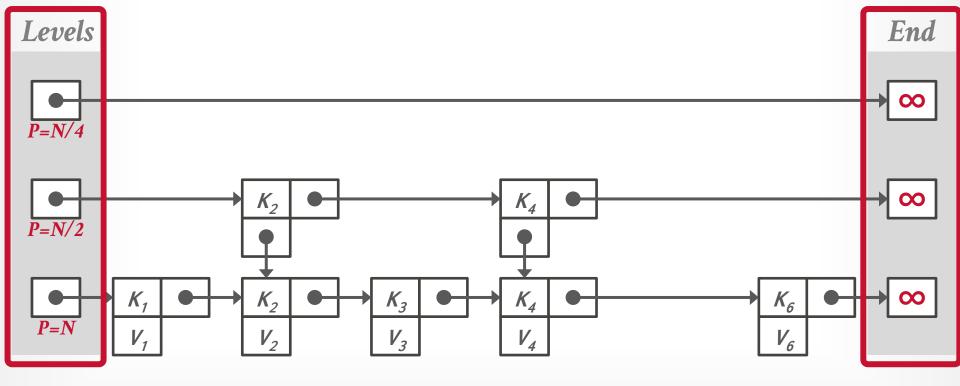
SingleStore



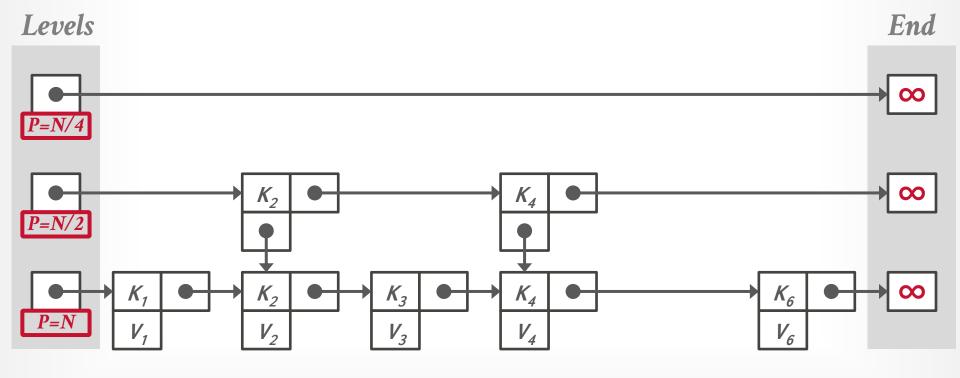




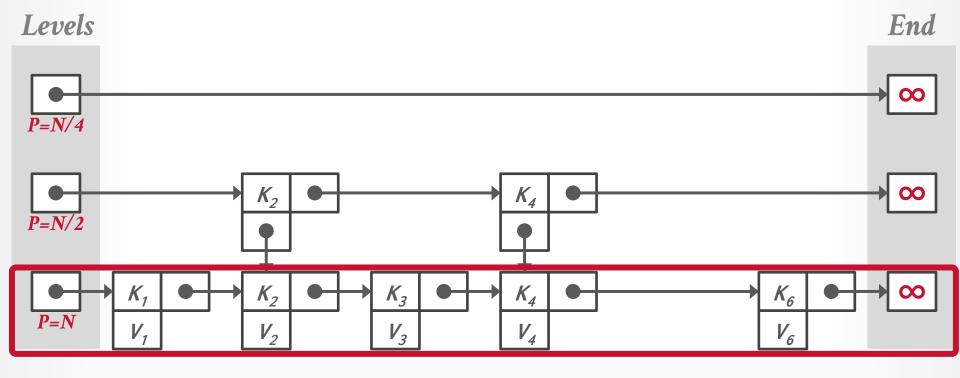




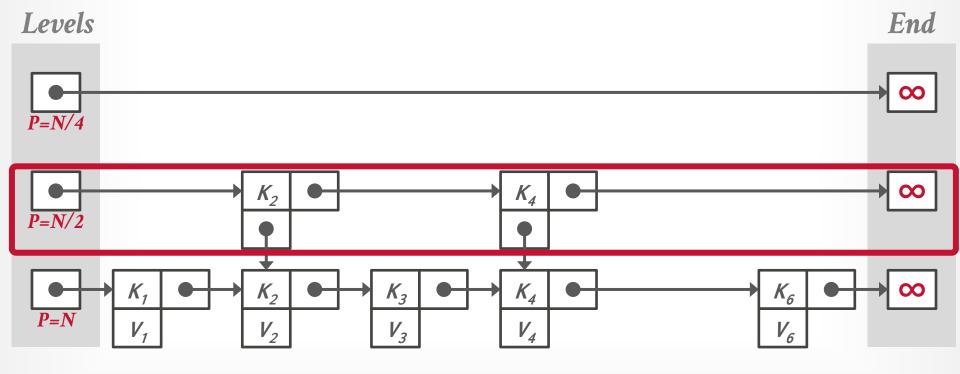




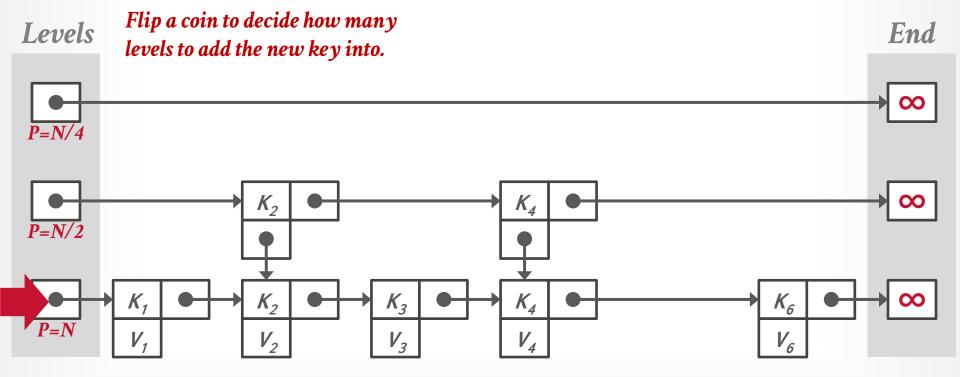




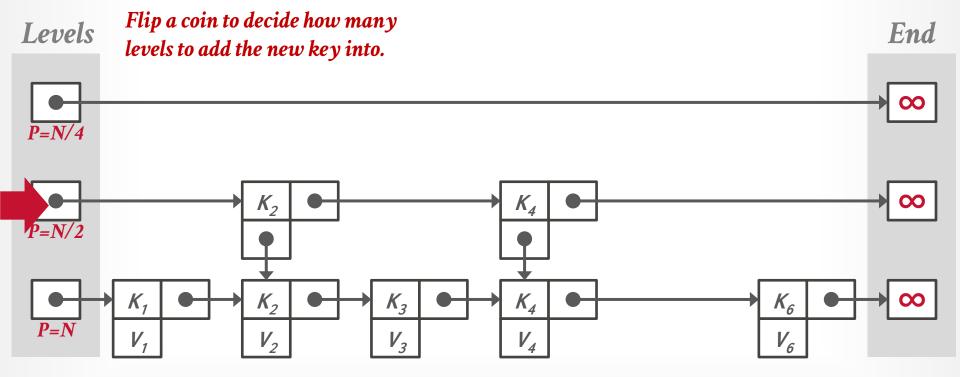




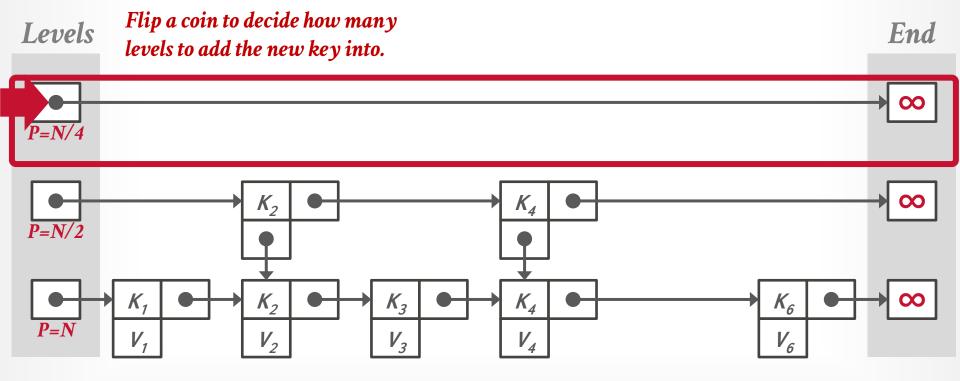




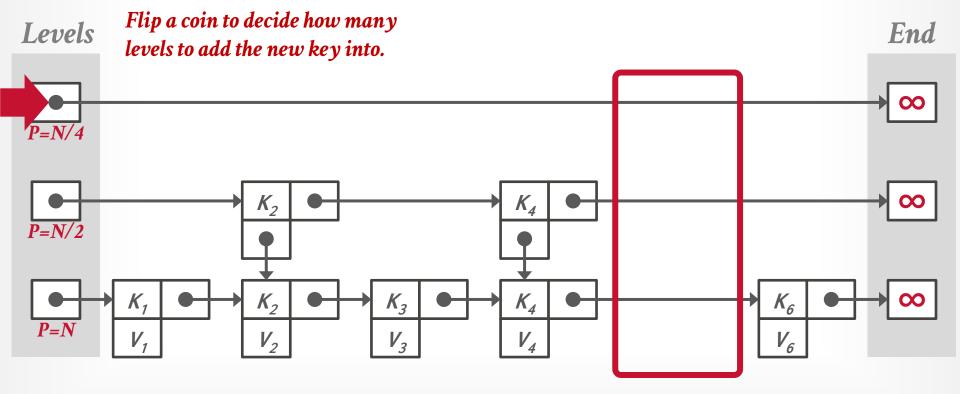




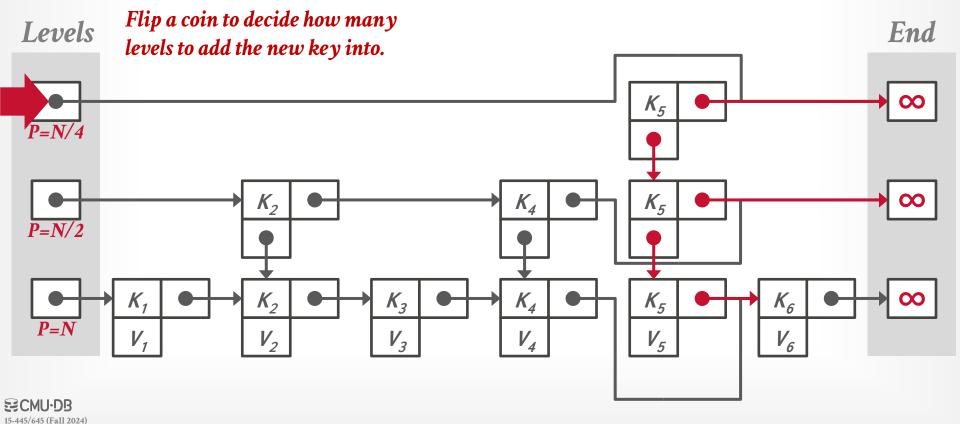


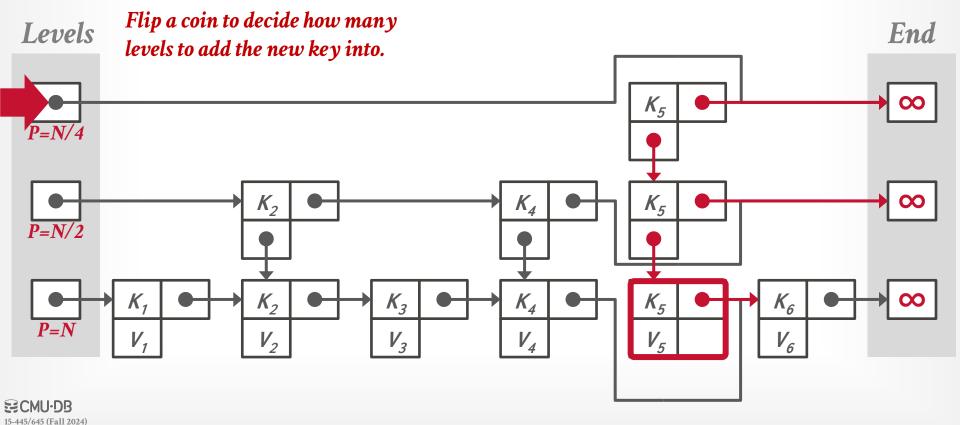


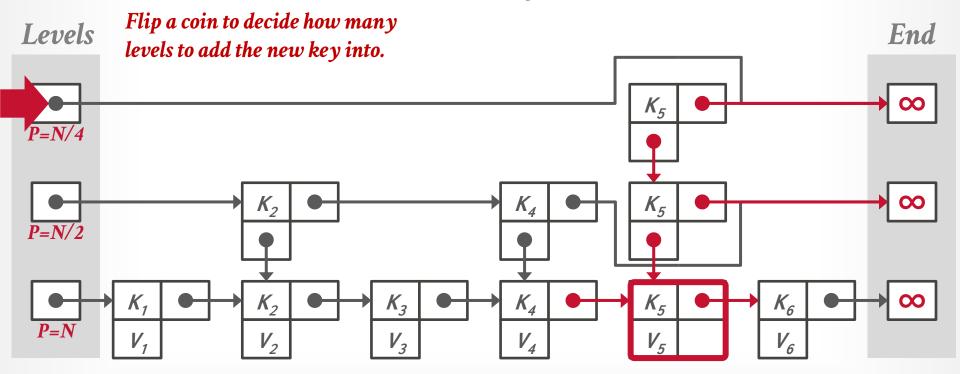




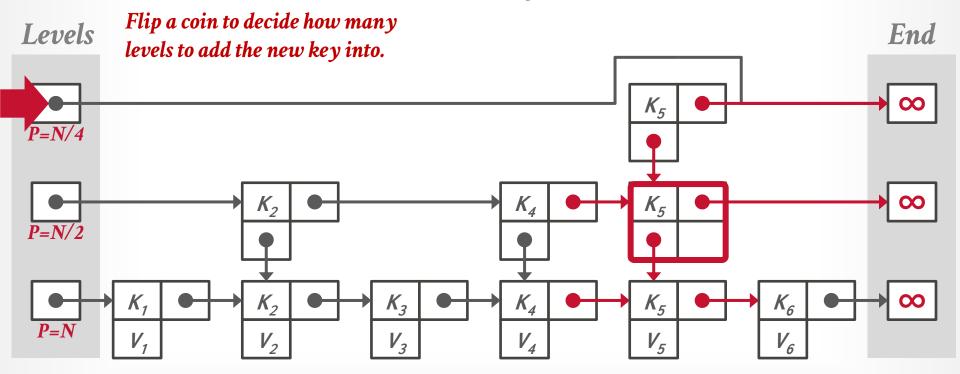




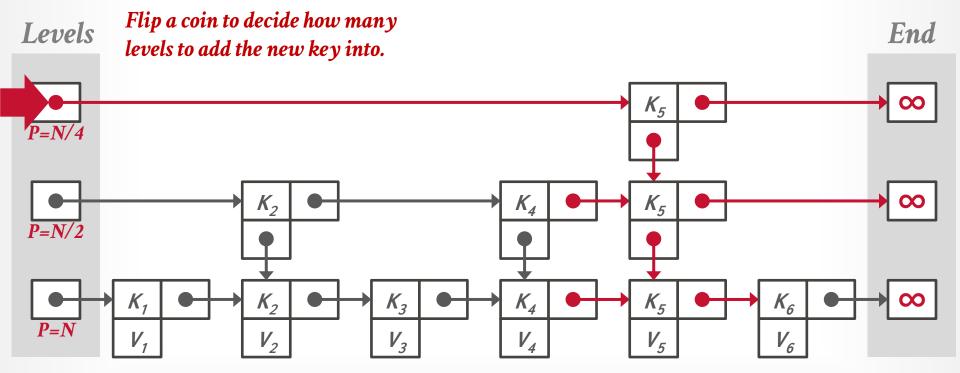




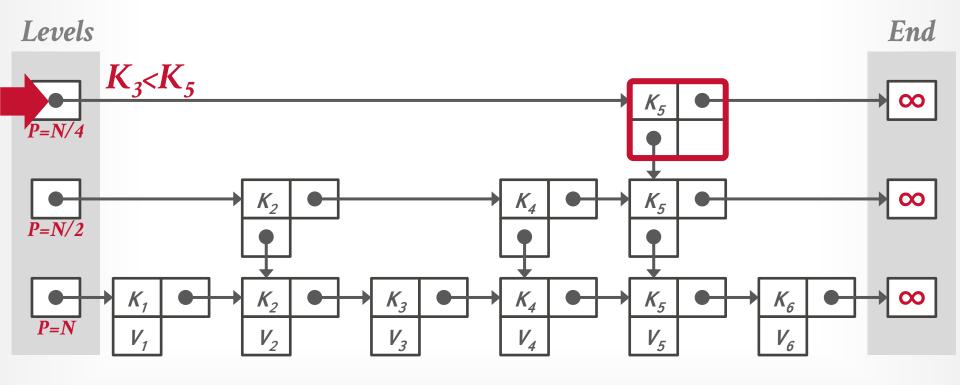




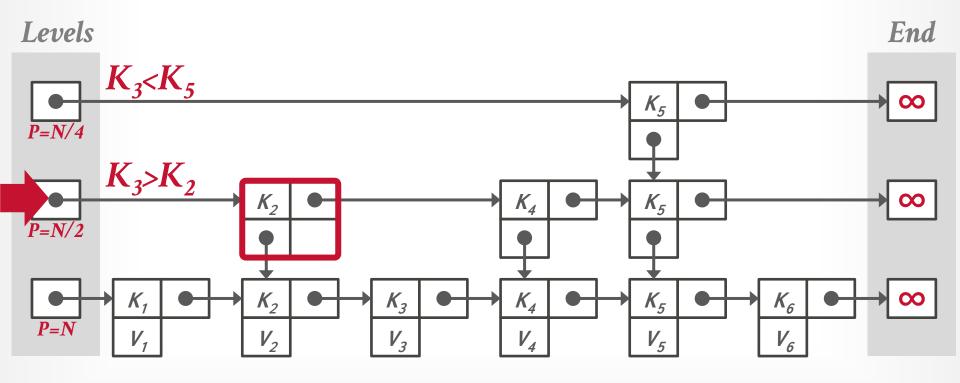




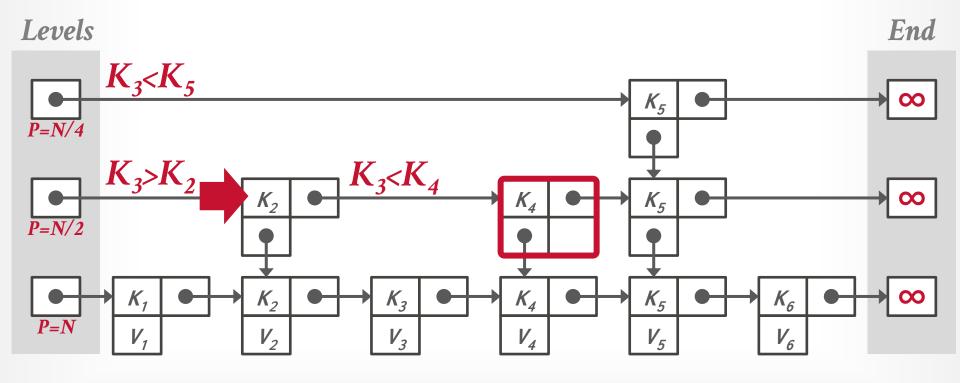




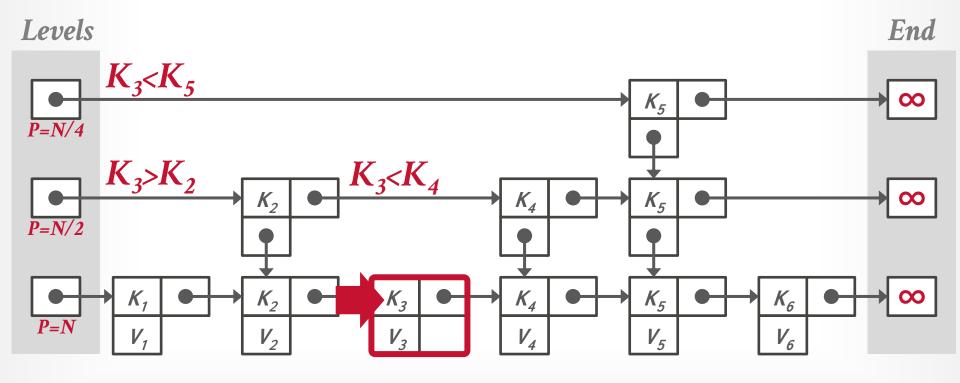














# SKIP LISTS: DELETE

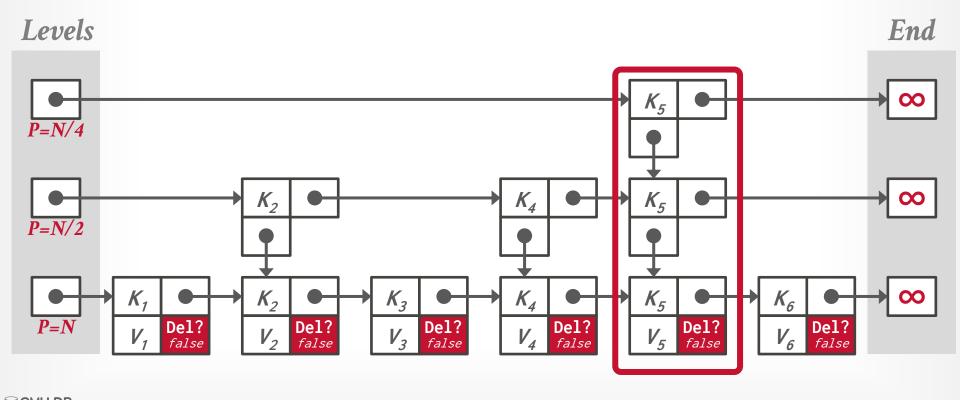
First <u>logically</u> remove a key from the index by setting a flag to tell threads to ignore.

Then **physically** remove the key once we know that no other thread is holding the reference.

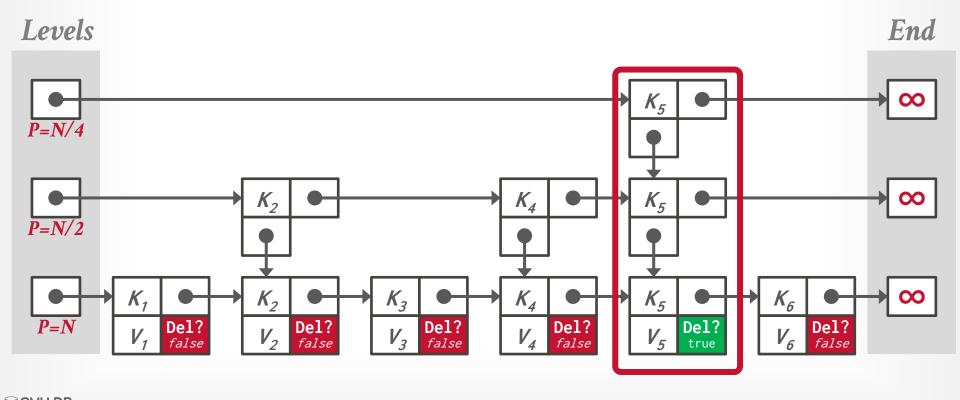


# SKIP LISTS: DELETE

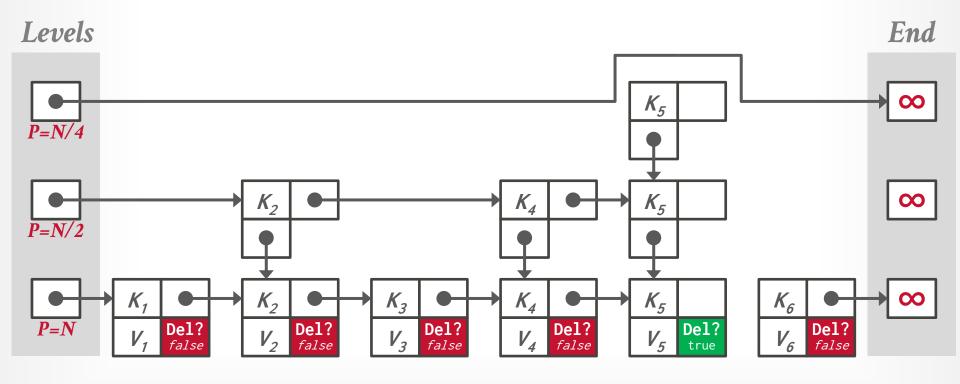
Delete K<sub>5</sub>



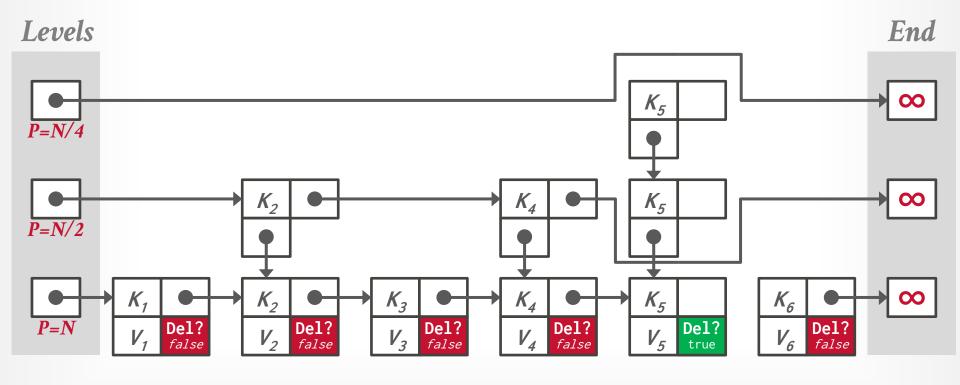




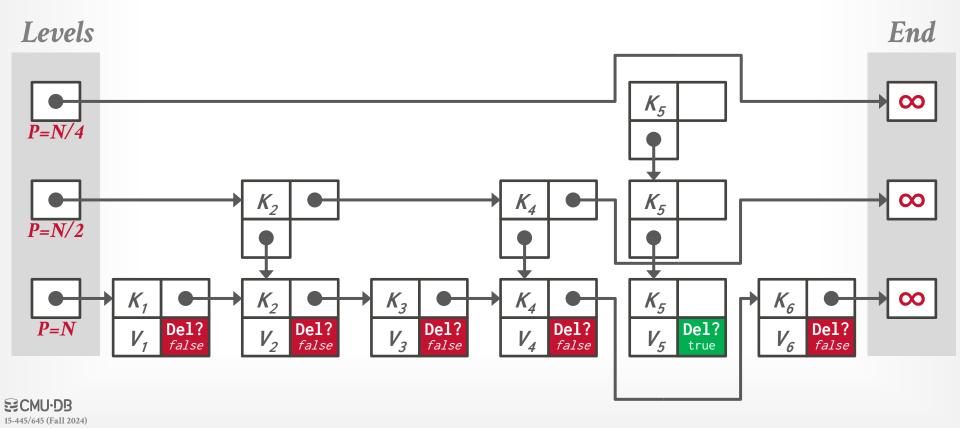


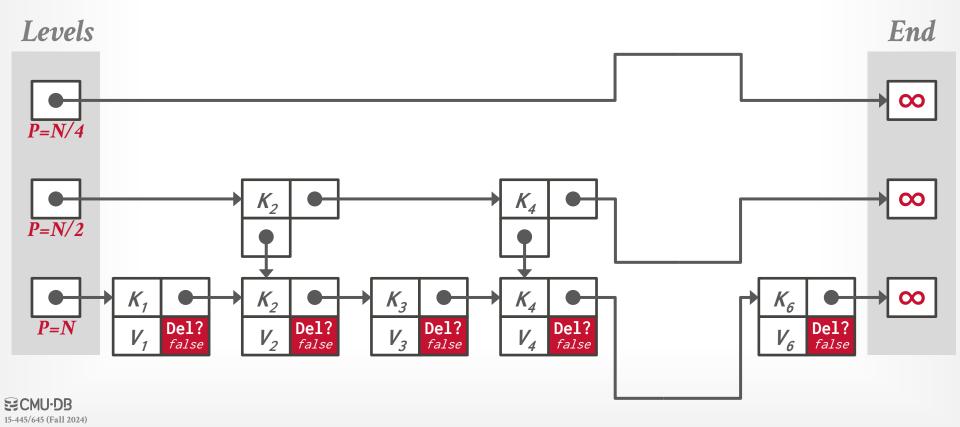












### SKIP LISTS

#### Advantages:

- → Uses less memory than a typical B+Tree if you do <u>not</u> include reverse pointers.
- → Insertions and deletions do not require rebalancing.

#### Disadvantages:

- → Not disk/cache friendly because they do not optimize locality of references.
- $\rightarrow$  Reverse search is non-trivial.



## **OBSERVATION**

The inner node keys in a B+Tree cannot tell you whether a key exists in the index. You must always traverse to the leaf node.

This means that you could have (at least) one buffer pool page miss per level in the tree just to find out a key does not exist.



### TRIE INDEX

Use a digital representation of keys to examine prefixes one-by-one.

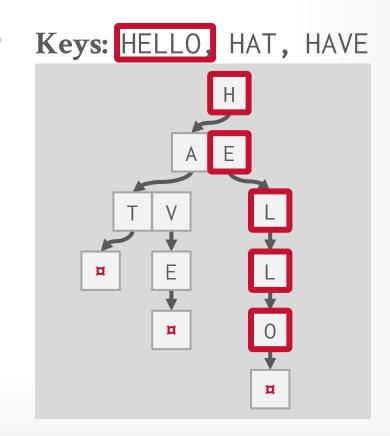
→ aka *Digital Search Tree*, *Prefix Tree*.

Shape depends on keys and lengths.

- → Does <u>not</u> depend on existing keys or insertion order.
- $\rightarrow$  Does <u>not</u> require rebalancing operations.

All operations have O(k) complexity where k is the length of the key.

- $\rightarrow$  Path to a leaf node represents a key.
- → Keys are stored implicitly and can be reconstructed from paths.





The **span** of a trie level is the number of bits that each partial key / digit represents.

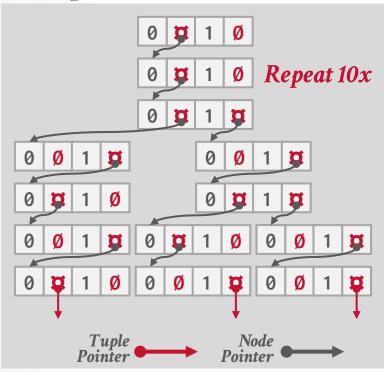
→ If the digit exists in the corpus, then store a pointer to the next level in the trie branch. Otherwise, store null.

This determines the <u>fan-out</u> of each node and the physical <u>height</u> of the tree.

 $\rightarrow$  *n*-way Trie = Fan-Out of *n* 



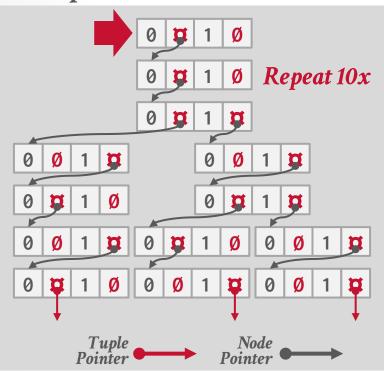
#### 1-bit Span Trie

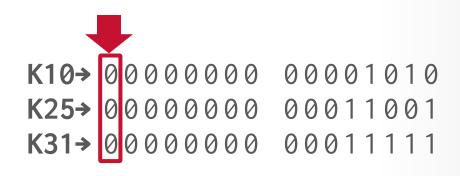


**Keys:** K10, K25, K31

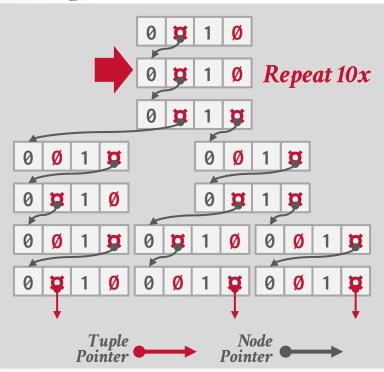
**K10→** 00000000 00001010 **K25→** 00000000 00011001 **K31→** 00000000 00011111

#### 1-bit Span Trie





#### 1-bit Span Trie



**Keys:** K10, K25, K31



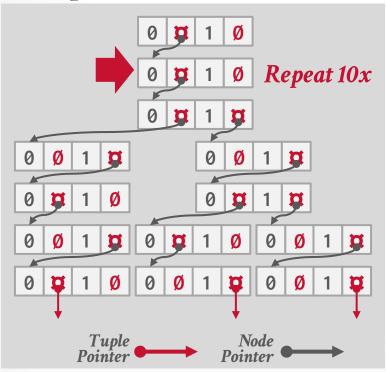
**K10→** 00000000 00001010

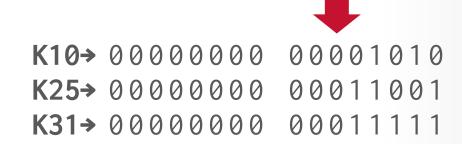
**K25→** 00000000 00011001

**K31→** 00000000 00011111



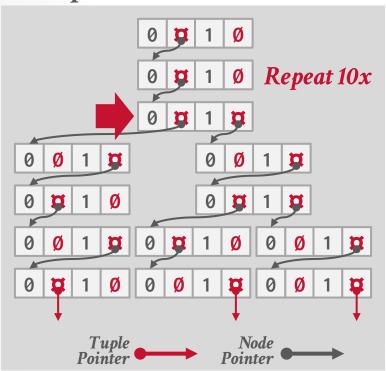
#### 1-bit Span Trie







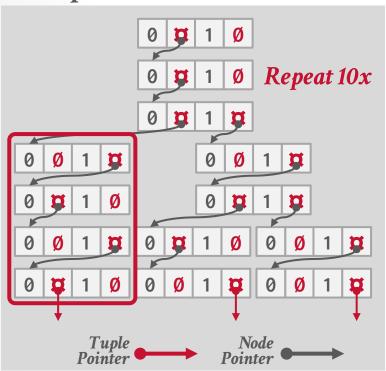
#### 1-bit Span Trie







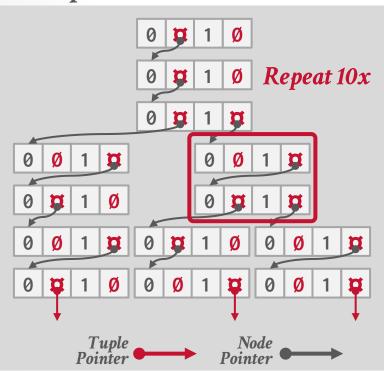
#### 1-bit Span Trie



```
K10→ 00000000 0000 1010 K25→ 00000000 00011001 K31→ 00000000 00011111
```

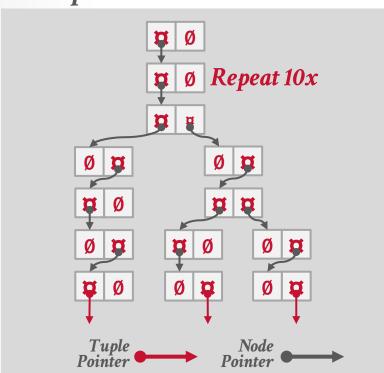


#### 1-bit Span Trie



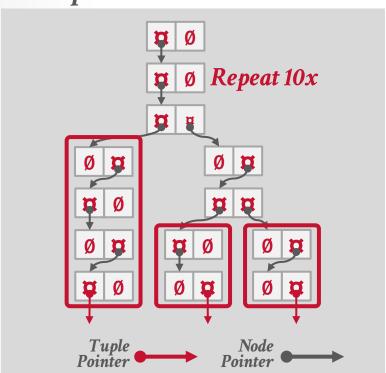
```
K10→ 00000000 00001010 K25→ 00000000 0001100 K31→ 0000000 000111111
```

#### 1-bit Span Trie



```
K10→ 00000000 00001010 K25→ 00000000 00011001 K31→ 0000000 00011111
```

#### 1-bit Span Trie

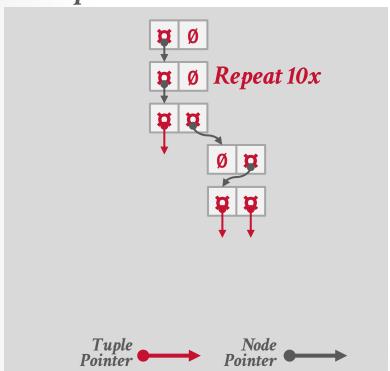


**Keys:** K10, K25, K31

**K10→** 00000000 00001010 **K25→** 00000000 00011001 **K31→** 0000000 00011111

#### RADIX TREE

#### 1-bit Span Radix Tree



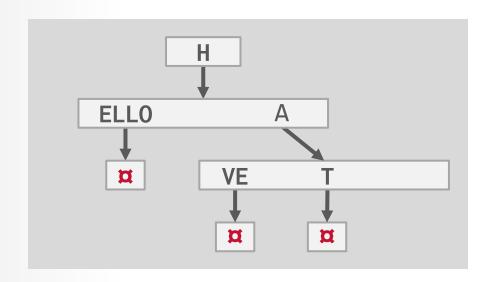
Vertically compressed trie that compacts nodes with a single child.

→ Also known as *Patricia Tree*.

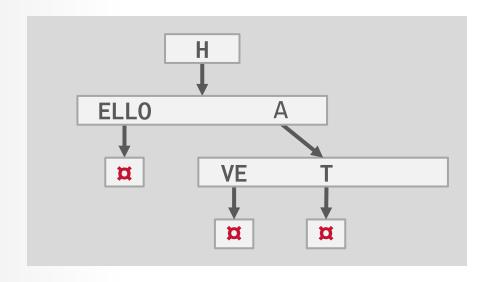
Can produce false positives, so the DBMS always checks the original tuple to see whether a key matches.





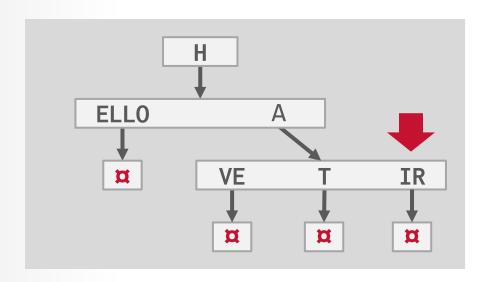






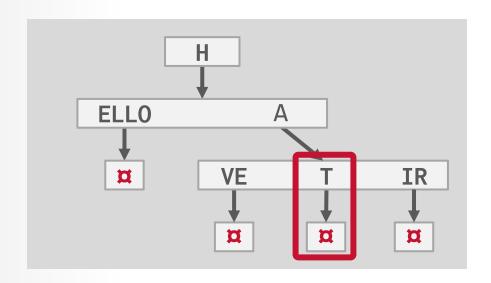
Insert HAIR





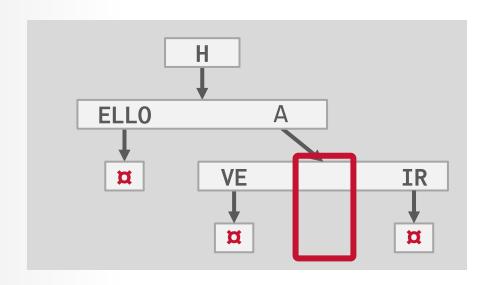
Insert HAIR





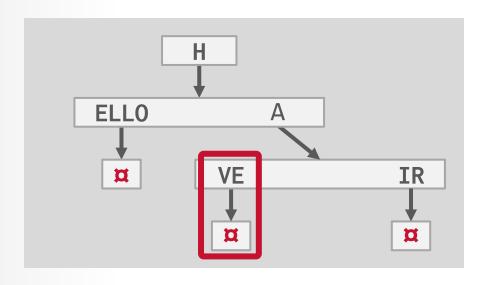
Insert HAIR
Delete HAT





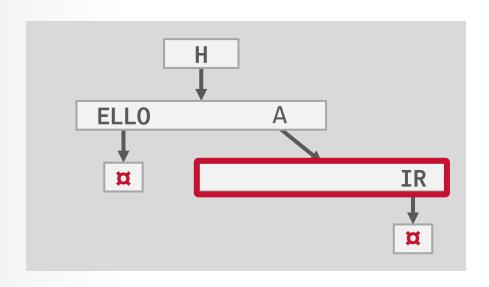
Insert HAIR
Delete HAT





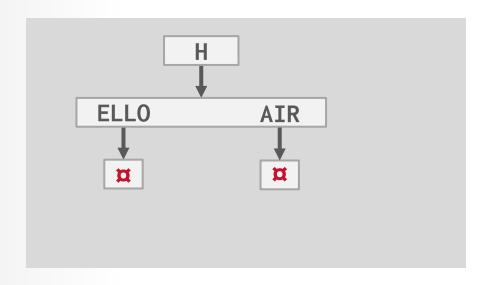
Insert HAIR
Delete HAT
Delete HAVE





Insert HAIR
Delete HAT
Delete HAVE





Insert HAIR
Delete HAT
Delete HAVE



## **OBSERVATION**

The indexes that we've discussed are useful for "point" and "range" queries:

- $\rightarrow$  Find all customers in the 15217 zipcode.
- → Find all orders between June 2024 and September 2024.

They are **not** good at keyword searches:

→ Example: Find all Wikipedia articles that contain the word "Pavlo"



### **OBSERVATION**

The indexes that we've discussed are useful for "point" and "range" queries:

- $\rightarrow$  Find all customers in the 15217 zipcode.
- → Find all orders between June 2024 and September 2024.

They are **not** good at keyword searches:

→ Example: Find all Wikipedia articles that contain the word "Pavlo"

#### revisions(id, content,...)

id	content
11	Wu-Tang Clan is an American hip hop musical collective formed in Staten Island, New York City, in 1992
22	Carnegie Mellon University (CMU) is a private research university in Pittsburgh, Pennsylvania. The institution was established in 1900 by Andrew Carnegie
33	In computing, a database is an organized collection of data or a type of data store based on the use of a database management system (DBMS), the software
44	Andrew Pavlo, best known as Andy Pavlo, is an associate professor of Computer Science at Carnegie Mellon University. He conducts research on database

CREATE INDEX idx\_rev\_cntnt
 ON revisions (content);

SELECT pageID FROM revisions
WHERE content LIKE '%Pavlo%';

### TNVFRTFD TNDFX

An **inverted index** stores a mapping of terms to records that contain those terms in the target attribute.

- → Sometimes called a *full-text search index*.
- → Originally called a *concordance* (1200s).

Many major DBMSs support these natively. But there are also specialized DBMSs and libraries.







Term /







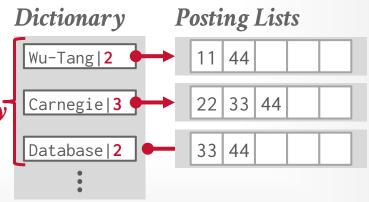


vespa splunk>

OpenSearch Sphinx

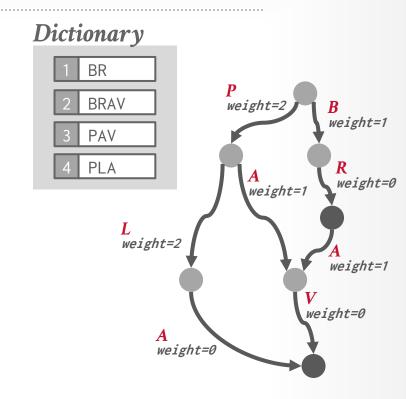
#### revisions(id, content,...)

	id	content
	11	Wu-Tang Clan is an American hip hop musical collective formed in Staten Island, New York City, in 1992
	22	Carnegie Mellon University (CMU) is a private research university in Pittsburgh, Pennsylvania. The institution was established in 1900 by Andrew Carnegie
	33	In computing, a database is an organized collection of data or a type of data store based on the use of a database management system (DBMS), the software
Ī	44	Andrew Pavlo, best known as Andy Pavlo, is an associate professor of Computer Science at Carnegie Mellon University. He conducts research on database



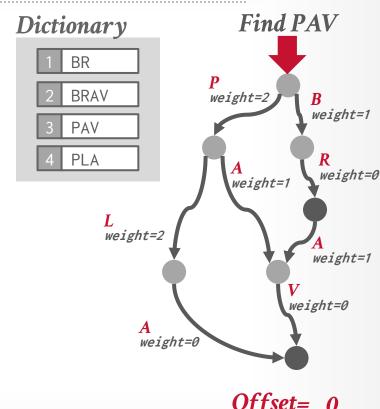
Uses a <u>Finite State Transducer</u> for determining offset of terms in dictionary.

- → Uses <u>compression methods</u> we previously discussed (e.g., delta, bit packing).
- → Also supports precomputed aggregations for terms and occurrences.



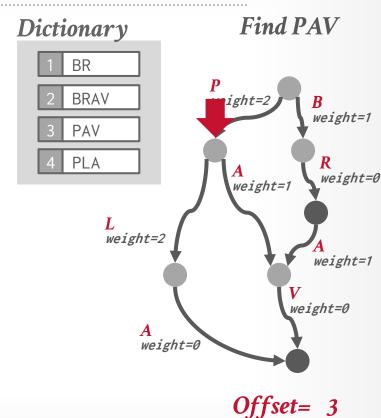
Uses a Finite State Transducer for determining offset of terms in dictionary.

- → Uses compression methods we previously discussed (e.g., delta, bit packing).
- → Also supports precomputed aggregations for terms and occurrences.



Uses a Finite State Transducer for determining offset of terms in dictionary.

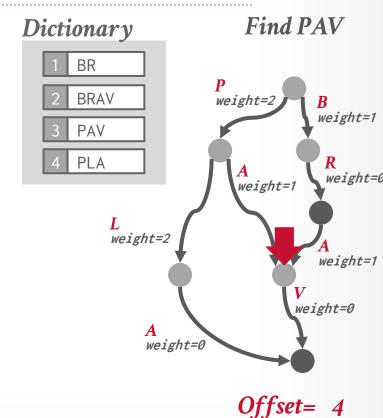
- → Uses compression methods we previously discussed (e.g., delta, bit packing).
- → Also supports precomputed aggregations for terms and occurrences.





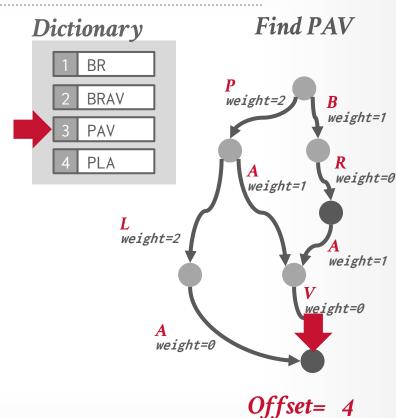
Uses a Finite State Transducer for determining offset of terms in dictionary.

- → Uses compression methods we previously discussed (e.g., delta, bit packing).
- → Also supports precomputed aggregations for terms and occurrences.



Uses a Finite State Transducer for determining offset of terms in dictionary.

- → Uses compression methods we previously discussed (e.g., delta, bit packing).
- → Also supports precomputed aggregations for terms and occurrences.





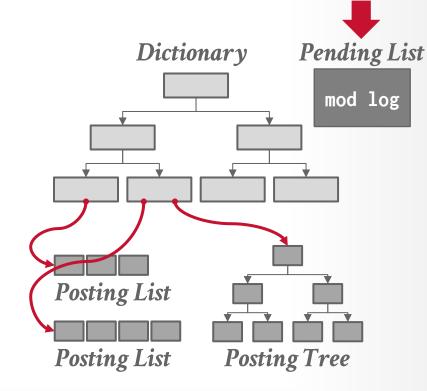
# INVERTED INDEX: POSTGRESQL

PostgreSQL's Generalized Inverted Index (GIN) uses a B+Tree for the term dictionary that map to a posting list data structure.

Posting list contents varies depending on number of records per term:

- → **Few**: Sorted list of record ids.
- → **Many**: Another B+Tree of record ids.

Uses a separate "pending list" log to avoid incremental updates.



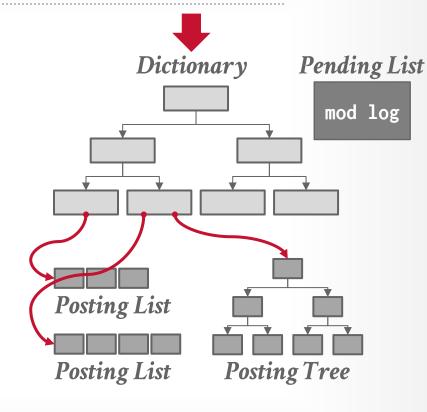
# INVERTED INDEX: POSTGRESQL

PostgreSQL's Generalized Inverted Index (GIN) uses a B+Tree for the term dictionary that map to a posting list data structure.

Posting list contents varies depending on number of records per term:

- → **Few**: Sorted list of record ids.
- → **Many**: Another B+Tree of record ids.

Uses a separate "pending list" log to avoid incremental updates.





## **OBSERVATION**

Inverted indexes search data based on its contents.

- → There is a little magic to tweak terms based on linguistic models.
- → Example: Normalization ("Wu-Tang" matches "Wu Tang").

Instead of searching for records containing exact keywords (e.g., "Wu-Tang"), an application may want search for records that are related to topics (e.g., "hip-hop groups with songs about slinging").



# VECTOR INDEXES

Specialized data structures to perform nearestneighbor searches on embeddings.

- $\rightarrow$  An embedding is an array of floating point numbers.
- → May also need to filter data before / after vector searches.

The correctness of a query depends on whether the result "feels right".



















## **VECTOR INDEXES**

Specialized data structures to perform nearestneighbor searches on embeddings.

- → An embedding is an array of floating point numbers.
- → May also need to filter data before / after vector searches.

The correctness of a query depends on whether the result "feels right".

















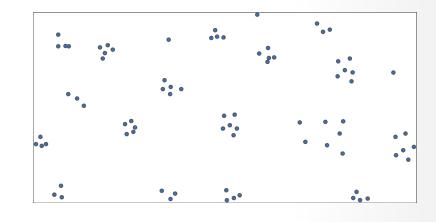


Partition vectors into smaller groups using a clustering algorithm.

To find a match, use same clustering algorithm to map into a group, then scan that group's vectors.

→ Also check nearby groups to improve accuracy.

Preprocess / quantize vectors to reduce dimensionality.

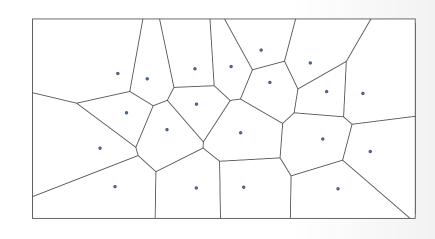


Partition vectors into smaller groups using a clustering algorithm.

To find a match, use same clustering algorithm to map into a group, then scan that group's vectors.

→ Also check nearby groups to improve accuracy.

Preprocess / quantize vectors to reduce dimensionality.



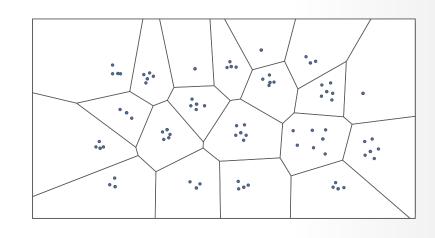


Partition vectors into smaller groups using a clustering algorithm.

To find a match, use same clustering algorithm to map into a group, then scan that group's vectors.

→ Also check nearby groups to improve accuracy.

Preprocess / quantize vectors to reduce dimensionality.

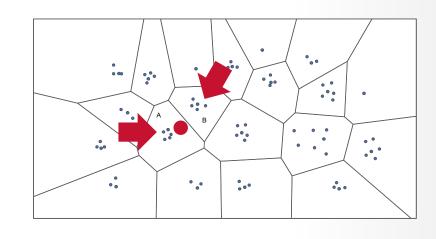


Partition vectors into smaller groups using a clustering algorithm.

To find a match, use same clustering algorithm to map into a group, then scan that group's vectors.

→ Also check nearby groups to improve accuracy.

Preprocess / quantize vectors to reduce dimensionality.



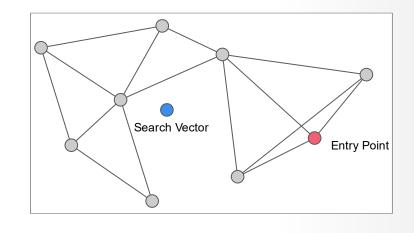


# VECTOR INDEXES: NAVIGABLE SMALL WORLDS

Build a graph where each node represents a vector and it has edges to its *n* nearest neighbors.

→ Can use multiple levels of graphs (HNSW)

To find a match for a given vector, enter the graph and then greedily choose the next edge that moves closer to that vector.



Example: Faiss, hnswlib

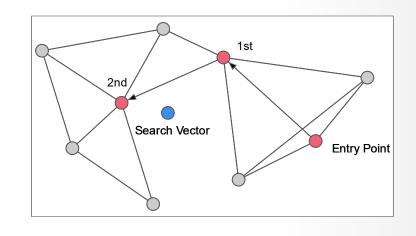


# VECTOR INDEXES: NAVIGABLE SMALL WORLDS

Build a graph where each node represents a vector and it has edges to its *n* nearest neighbors.

→ Can use multiple levels of graphs (HNSW)

To find a match for a given vector, enter the graph and then greedily choose the next edge that moves closer to that vector.



Example: Faiss, hnswlib



#### CONCLUSION

We will see filters again this semester.

B+Trees are still the way to go for tree indexes.

Inverted indexes are covered in CMU 11-442.

We did not discuss geo-spatial tree indexes:

- → Examples: R-Tree, Quad-Tree, KD-Tree
- $\rightarrow$  This is covered in CMU 15-826.



# **NEXT CLASS**

How to make indexes thread-safe!

