CARNEGIE MELLON UNIVERSITY
COMPUTER SCIENCE DEPARTMENT
15-445/645 – DATABASE SYSTEMS (SPRING 2024)
PROF. JIGNESH PATEL

Homework #2 (by Shivang and Avery) – Solutions

Due: **Friday February 16, 2024 @ 11:59pm**

IMPORTANT:

- Enter all of your answers into **Gradescope by 11:59pm on Friday February 16, 2024.**
- **Plagiarism**: Homework may be discussed with other students, but all homework is to be completed **individually**.

For your information:

- Graded out of **100** points; **4** questions total
- Rough time estimate: ≈4-6 hours (1-1.5 hours for each question)

Revision : 2024/02/20 17:29

Question	Points	Score
Storage Models	24	
Linear Hashing and Cuckoo Hashing	24	
Extendible Hashing	20	
B+Tree	32	
Total:	100	

Question 1: Storage Models . . . . [24 points]

Graded by:

Consider a database with a single table `T(song_id,song_name,artist_id,duration number_of_streams)`, where `song_id` is the *primary key*, and all attributes are the same fixed width. Suppose `T` has 10,000 tuples that fit into 500 pages, Ignore any additional storage overhead for the table (e.g., page headers, tuple headers). Additionally, you should make the following assumptions:

- The DBMS does *not* have any additional meta-data (e.g., sort order, zone maps).
- `T` does *not* have any indexes (including for primary key `song_id`).
- None of `T`'s pages are already in the buffer pool. The DMBS has an infinite buffer pool.
- Content-wise, the tuples of `T` will always make each query run the longest possible and do the most page accesses.
- The tuples of `T` can be in any order (keep this in mind when computing *minimum* versus *maximum* number of pages that the DBMS will potentially have to read and think of all possible orderings)

(a) Consider the following query:

```
SELECT MAX(number_of_streams) FROM T
    WHERE duration > 100 AND artist_id == 123321 ;
```

i. **[3 points]** Suppose the DBMS uses the decomposition storage model (DSM) with implicit offsets. How many pages will the DBMS potentially have to read from disk to answer this query? (Keep in mind our assumption about the contents of `T`!)
☐ 1-100 ☐ 101-200 [✓ **201-300**] ☐ 301-500 ☐ $\geq$ 501 ☐ Not possible to determine

Solution: 300 pages. There are 100 pages per attribute. 100 pages to find `duration` and another 100 to find `artist_id` for all tuples. In the worst-case scenario for `T`'s content, `number_of_streams` for all tuples must be accessed as well. Hence, another 100 pages must be read.

ii. **[3 points]** Suppose the DBMS uses the N-ary storage model (NSM). How many pages will the DBMS potentially have to read from disk to answer this query? (Keep in mind our assumption about the contents of `T`!)
☐ 1-100 ☐ 101-200 ☐ 201-300 [✓ **301-500**] ☐ $\geq$ 501 ☐ Not possible to determine

Solution: 500 pages. To find `duration` and `artist_id` for all tuples, all pages must be accessed.

(b) Now consider the following query:

```
SELECT song_name, artist_id FROM T
    WHERE song_id = 15445 OR song_id = 15645 OR song_id = 15721;
```

i. Suppose the DBMS uses the decomposition storage model (DSM) with implicit offsets.

α) **[3 points]** What is the *minimum* number of pages that the DBMS will potentially have to read from disk to answer this query?
☐ 1 [✓ **2-4**] ☐ 5-100 ☐ 101-200 ☐ 201-500 ☐ $\geq$ 501
☐ Not possible to determine

Solution: 3 pages. Suppose all three primary keys appear on the first page. Since all attributes are of the same fixed width, each attribute of `song_id=15445` and `song_id=15645` will also appear on the same page. We'll thus need to read 1 page to find the three primary keys and read 2 pages to access `song_name`, `artist_id` at their corresponding offsets.

β) **[3 points]** What is the *maximum* number of pages that the DBMS will potentially have to read from disk to answer this query?
☐ 1 ☐ 2-4 ☐ 5-100 [✓ **101-200**] ☐ 201-500 ☐ $\geq$ 501
☐ Not possible to determine

Solution: 106 pages. There are 100 pages per attribute. In the worst case, we scan through all 100 pages to find the three primary keys. In the worst case, three two primary keys will be located on different pages. Since all attributes are of the same fixed width, each attribute of `song_id=15445`, `song_id=15645` and `song_id=15721` will also appear on different pages. Hence we must read 3 pages to access each attribute at their corresponding offsets. Thus, we read 6 pages in total to access `song_name`, `artist_id`.

ii. Suppose the DBMS uses the N-ary storage model (NSM).

α) **[3 points]** What is the *minimum* number of pages that the DBMS will potentially have to read from disk to answer this query?
[✓ **1**] ☐ 2-4 ☐ 5-100 ☐ 101-200 ☐ 201-500 ☐ $\geq$ 501
☐ Not possible to determine

Solution: We find the tuples of all three primary keys on the first page. No need to look in other pages since all attributes are stored together.

β) **[3 points]** What is the *maximum* number of pages that the DBMS will potentially have to read from disk to answer this query?
☐ 1 ☐ 2-4 ☐ 5-100 ☐ 101-200 [✓ **201-500**] ☐ $\geq$ 501
☐ Not possible to determine

Question 1 continues…

Solution: 500 pages. At least one tuple with matching primary key is located on the last page. We must thus scan through every page.

(c) Finally consider the following query:

```
SELECT song_id, number_of_streams FROM T
    WHERE duration = (SELECT MIN(duration) FROM T);
```

Suppose the DBMS uses the decomposition storage model (DSM) with implicit offsets.

i. **[3 points]** What is the *minimum* number of pages that the DBMS will potentially have to read from disk to answer this query?
☐ 1 ☐ 2-5 ☐ 100-200 ☐ 201-299 [✓ $\geq$ 300] ☐ Not possible to determine

Solution: 300 pages. 100 pages for the inner select, and 100 pages to get the song_id and number_of_streams since the buffer pool will have the duration pages from the inner select. Remember content-wise the tuples make the queries always run for the longest time, you can only consider different orderings of the tuples.

ii. **[3 points]** What is the *maximum* number of pages that the DBMS will potentially have to read from disk to answer this query?
☐ 1 ☐ 2-5 ☐ 100-200 ☐ 201-299 [✓ $\geq$ 300] ☐ Not possible to determine

Solution: 300 pages. 100 pages for the inner select, and 100 pages to get the song_id and number_of_streams since the buffer pool will have the duration pages from the inner select. Remember content-wise the tuples make the queries always run for the longest time, you can only consider different orderings of the tuples

Question 2: Linear Hashing and Cuckoo Hashing . . . . [24 points]

Graded by: Consider the following linear probe hashing schema:

1. The table have a size of 4 slots, each slot can only contain one key value pair.
2. The hashing function is
   $h_1(\texttt{x}) = \texttt{x \% 4}$.
3. When there is conflict, it finds the next free slot to insert key value pairs.
4. The original table is empty.

(a) Insert key value pair (3, A) and (4, B). (for (3,A), 3 is the key and A is the value) Select the value in each entry of the resulting table.

i. **[1 point]** Entry 0 (key % 4 = 0) ☐ A [✓ **B**] ☐ Empty
ii. **[1 point]** Entry 1 (key % 4 = 1) ☐ A ☐ B [✓ **Empty**]
iii. **[1 point]** Entry 2 (key % 4 = 2) ☐ A ☐ B [✓ **Empty**]
iv. **[1 point]** Entry 3 (key % 4 = 3) [✓ **A**] ☐ B ☐ Empty

Solution: A is inserted into Entry 3, B is inserted into Entry 0.

(b) After the changes from part **(a)**, insert key value pair (8, C) and then (9, D). Select the value in each entry of the resulting table.

i. **[1 point]** Entry 0 (key % 4 = 0) ☐ A [✓ **B**] ☐ C ☐ D ☐ Empty
ii. **[1 point]** Entry 1 (key % 4 = 1) ☐ A ☐ B [✓ **C**] ☐ D ☐ Empty
iii. **[1 point]** Entry 2 (key % 4 = 2) ☐ A ☐ B ☐ C [✓ **D**] ☐ Empty
iv. **[1 point]** Entry 3 (key % 4 = 3) [✓ **A**] ☐ B ☐ C ☐ D ☐ Empty

Solution: C is tried to be inserted into entry 0, but due to the conflict, it is inserted into the next free slot which is entry 1. D is tried to be inserted into entry 1, but as it is occupied by C, D is inserted into entry 2.

Question 2 continues. . .

Consider the following cuckoo hashing schema:

1. Both tables have a size of 4.
2. The hashing function of the first table returns the fourth and third least significant bits: $h_1(\texttt{x}) =$ `(x >> 2) & 0b11`.
3. The hashing function of the second table returns the least significant two bits: $h_2(\texttt{x}) =$ `x & 0b11`.
4. When inserting, try table 1 first.
5. When replacement is necessary, first select an element in the second table.
6. The original entries in the table are shown in the figure below.

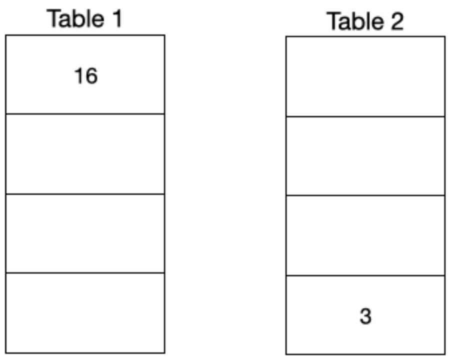


Figure 1: Initial contents of the hash tables.

(a) Insert key 32 and then delete 16. Select the value in each entry of the resulting two tables.

i. Table 1

α)	**[1 point]**	Entry 0 (`0b00`)	☐ 32	☐ 16	[✓ **Empty**]
β)	**[1 point]**	Entry 1 (`0b01`)	☐ 32	☐ 16	[✓ **Empty**]
γ)	**[1 point]**	Entry 2 (`0b10`)	☐ 32	☐ 16	[✓ **Empty**]
δ)	**[1 point]**	Entry 3 (`0b11`)	☐ 32	☐ 16	[✓ **Empty**]

ii. Table 2

α)	**[1 point]**	Entry 0 (`0b00`)	[✓ **32**]	☐ 3	☐ Empty
β)	**[1 point]**	Entry 1 (`0b01`)	☐ 32	☐ 3	[✓ **Empty**]
γ)	**[1 point]**	Entry 2 (`0b10`)	☐ 32	☐ 3	[✓ **Empty**]
δ)	**[1 point]**	Entry 3 (`0b11`)	☐ 32	[✓ **3**]	☐ Empty

Question 2 continues…

Solution: 32 tries to insert into table 1 first but due to the conflict, it inserts into the first entry of table 2. 16 is then deleted makes table 1 an empty table.

(b) After the changes from part **(a)**, insert key 27, then delete 3, then insert key 8. Select the value in each entry of the resulting two tables.

i. Table 1

α)	**[1 point]**	Entry 0 (`0b00`)	☐ 8	[✓ **32**]	☐ 27	☐ Empty
β)	**[1 point]**	Entry 1 (`0b01`)	☐ 8	☐ 32	☐ 27	[✓ **Empty**]
γ)	**[1 point]**	Entry 2 (`0b10`)	☐ 8	☐ 32	[✓ **27**]	☐ Empty
δ)	**[1 point]**	Entry 3 (`0b11`)	☐ 8	☐ 32	☐ 27	[✓ **Empty**]

ii. Table 2

α)	**[1 point]**	Entry 0 (`0b00`)	☐ 32	[✓ **8**]	☐ 27	☐ Empty
β)	**[1 point]**	Entry 1 (`0b01`)	☐ 32	☐ 8	☐ 27	[✓ **Empty**]
γ)	**[1 point]**	Entry 2 (`0b10`)	☐ 32	☐ 8	☐ 27	[✓ **Empty**]
δ)	**[1 point]**	Entry 3 (`0b11`)	☐ 32	☐ 8	☐ 27	[✓ **Empty**]

Solution: 27 is inserted into table 1 `0b10` based on h_2. 3 is deleted. 8 tries to insert into table 1 `0b10` but there's conflict. 8 is then inserted into table 2 `0b00`. 32 is replaced and rehashed to table 1 `0b00`.

Question 3: Extendible Hashing .... [20 points]

Graded by:

Consider an extendible hashing structure such that:

- Each bucket can hold up to two records.
- The hashing function uses the lowest g bits, where g is the global depth.
- A new extendible hashing structure is initialized with $g = 0$ and one empty bucket
- If multiple keys are provided in a question, assume they are inserted one after the other from left to right.

(a) Starting from an empty table, insert keys 0, 3.

i. **[1 point]** What is the global depth of the resulting table?

[✓ **0**] ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ None of the above

Solution: No split has occurred yet because the first bucket (on initialization) can hold 2 arbitrary values. Thus global depth is same as its initial value of 0.

ii. **[1 point]** What is the local depth of the bucket containing 0?

[✓ **0**] ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ None of the above

Solution: There is only one bucket (created on initialization), and it holds both 0 and 3. Since no split has occurred yet, the bucket has local depth $d = 0$.

(b) Starting from the result in **(a)**, you insert keys 23, 33.

i. **[2 points]** What is the global depth of the resulting table?

☐ 0 ☐ 1 [✓ **2**] ☐ 3 ☐ 4 ☐ None of the above

Solution: Two total splits occurred when inserting 23 and 33. After these splits: The table looks like the following:
Global depth = 2
`b0,b2` = 0 // at local depth 1
`b1` = 33 // at local depth 2
`b3` = 3,23 // at local depth 2

ii. **[1 point]** What is the local depth of the bucket containing 0?

☐ 0 [✓ **1**] ☐ 2 ☐ 3 ☐ 4 ☐ None of the above

Solution: See the above table.

(c) Starting from the result in **(b)**, you insert keys 13, 27.

i. **[2 points]** What is the global depth of the resulting table?

☐ 0 ☐ 1 ☐ 2 [✓ **3**] ☐ 4 ☐ None of the above

Solution: 13 inserts into `b1` without causing a split. 27 causes a split in `b3`. The updated table looks as follows: Global depth = 3
`b0,b2,b4,b6` = 0 // at local depth 1
`b1,b5` = 33,13 // at local depth 2

Question 3 continues…

b3 = 3,27 // at local depth 3
b7 = 23 // at local depth 3

ii. **[2 points]** What is the local depth of the bucket containing 13?
☐ 0 ☐ 1 [✓ **2**] ☐ 3 ☐ 4 ☐ None of the above

Solution: See the above table.

iii. **[1 point]** What is the local depth of the bucket containing 3?
☐ 0 ☐ 1 ☐ 2 [✓ **3**] ☐ 4 ☐ None of the above

Solution: See the above table.

iv. **[1 point]** Which value, if inserted, will hash to the same bucket as the bucket containing key 27?
[✓ **43**] ☐ 4 ☐ 21 ☐ 13 ☐ None of the above

Solution: Only 43 will hash to `b3`.

(d) **[3 points]** Starting from the result in **(c)**, which **key(s)**, if inserted next, will **not** cause a split?
☐ 35 ☐ 9 ☐ 17 ☐ 41 [✓ **16**] ☐ None of the above

Solution: To avoid a split in the current table the value must map to one of the following: `b0,b2,b4,b6,b7`. Out of the options provided only 16 hashs to one of these.

(e) **[3 points]** Starting from the result in **(c)**, which **key(s)**, if inserted next, will cause a split and increase the table's global depth?
☐ 0 ☐ 2 [✓ **51**] ☐ 9 ☐ 1 ☐ None of the above

Solution: To cause a split while doubling the table size, one must insert a key that hashes to `b3`, since it is the only full bucket whose local depth is equal to the global depth. Out of the options only 51 maps to this bucket.

(f) **[3 points]** Starting from an empty table, insert keys 64, 128, 256, 512. What is the global depth of the resulting table?
☐ 4 ☐ 5 ☐ 6 ☐ 7 [✓ **8**] ☐ $\geq$9

Solution: Since each bucket can hold at most two keys, three or more keys cannot hash to the same bucket without causing splits. Keys 128, 256 and 512 share the same lower 7 bits. Hence, the table will split until global depth reaches $g = 8$. When $g = 8$, 128 will no longer be hashed to the same bucket as 512 and 256.

Question 4: B+Tree ....[32 points]

Graded by:

Consider the following B+tree.

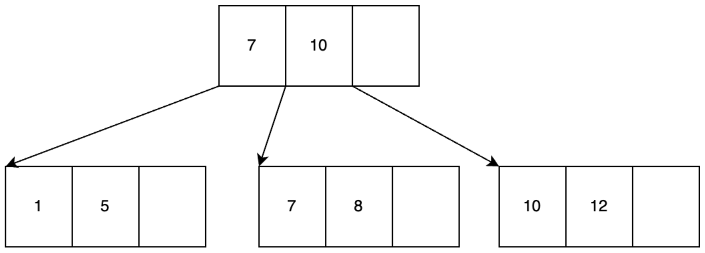


Figure 2: B+ Tree of order $d = 4$ and height $h = 2$.

When answering the following questions, be sure to follow the procedures described in class and in your textbook. You can make the following assumptions:

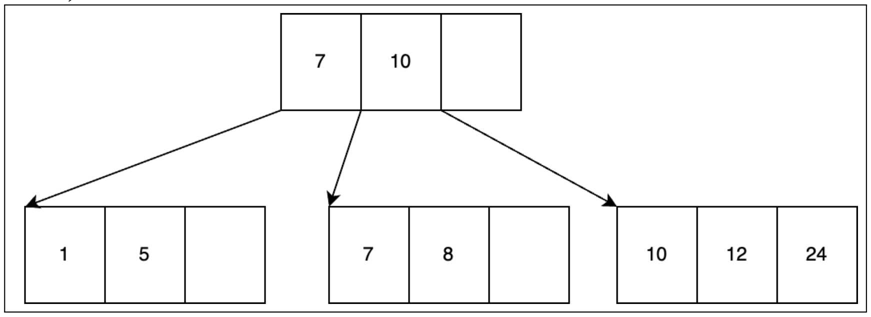
- A left pointer in an internal node guides towards keys $<$ than its corresponding key, while a right pointer guides towards keys $\geq$.
- A leaf node underflows when the number of **keys** goes below $\lceil \frac{d-1}{2} \rceil$.
- An internal node underflows when the number of **pointers** goes below $\lceil \frac{d}{2} \rceil$.

Note that B+ tree diagrams for this problem omit leaf pointers for convenience. The leaves of actual B+ trees are linked together via pointers, forming a singly linked list allowing for quick traversal through all keys.

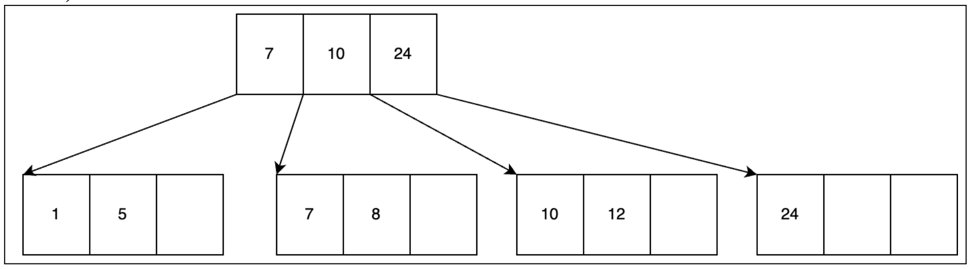
Question 4 continues...

(a) **[4 points]** Insert 24^* into the B+tree. Select the resulting tree.

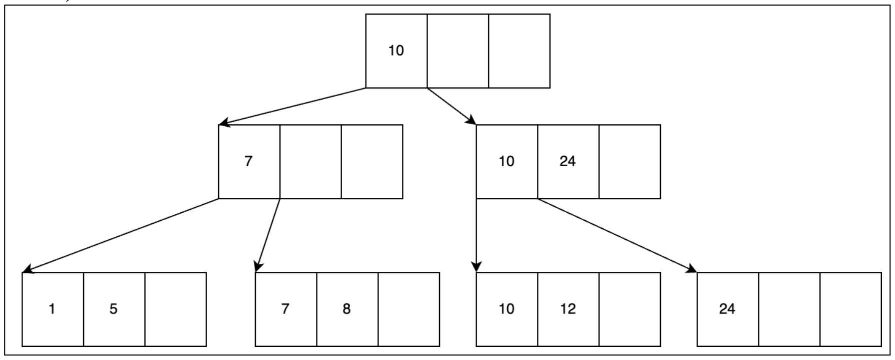
■ **A)**



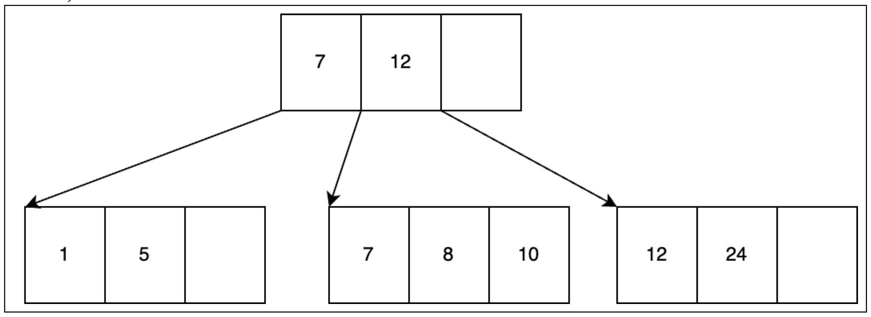
☐ B)



☐ C)



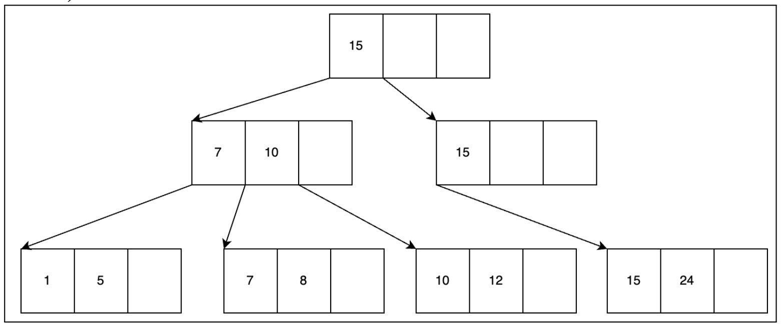
☐ D)



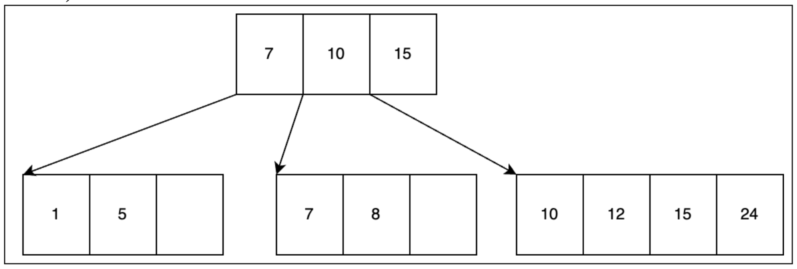
Solution: Inserting 24^* adds one element in the right-most leaf, as d = 4, it should not cause any splits or merges.

(b) **[4 points]** Starting with the tree that results from **(a)**, insert 15^*. Select the resulting tree.

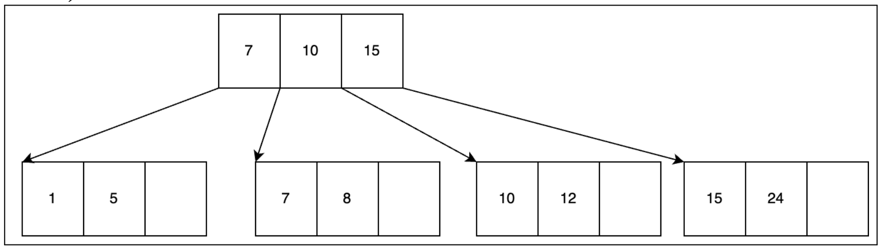
☐ A)



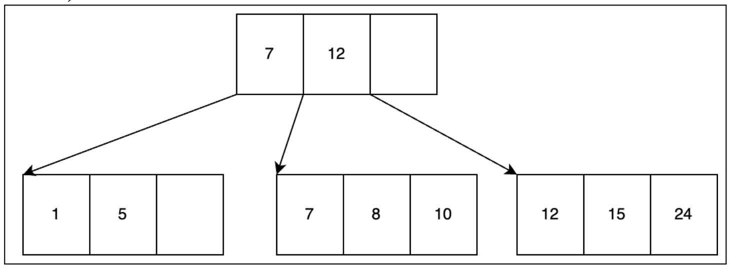
☐ B)



[✓ **C)**]



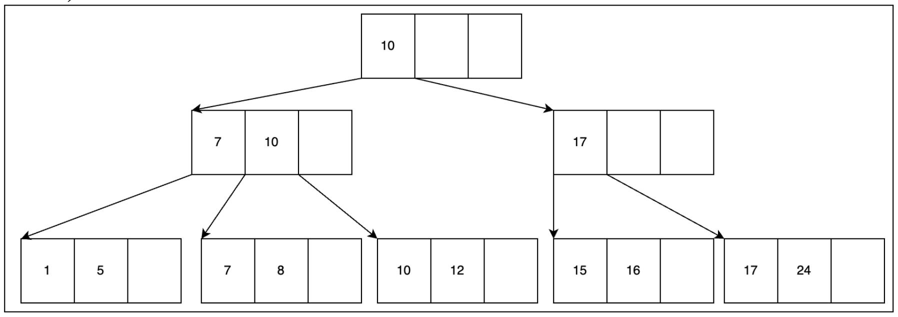
☐ D)



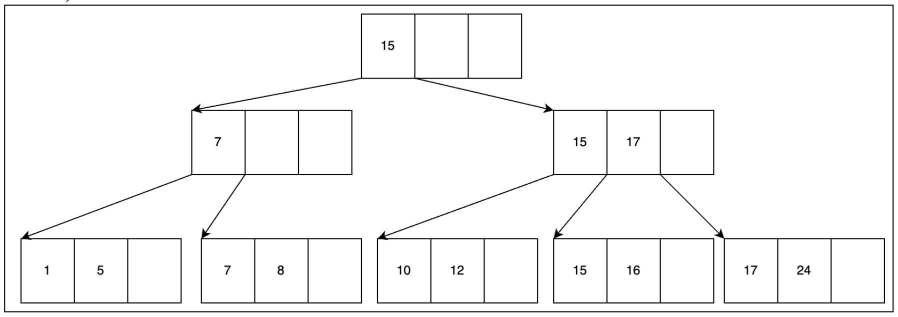
Solution: Inserting 15^* causes the right-most leaf node to split. And as the root level node still have space, it should not cause recursive splits.

(c) **[5 points]** Starting with the tree that results from **(b)**, insert 16^* and then insert 17^*. Select the resulting tree (the result after inserting 17^*).

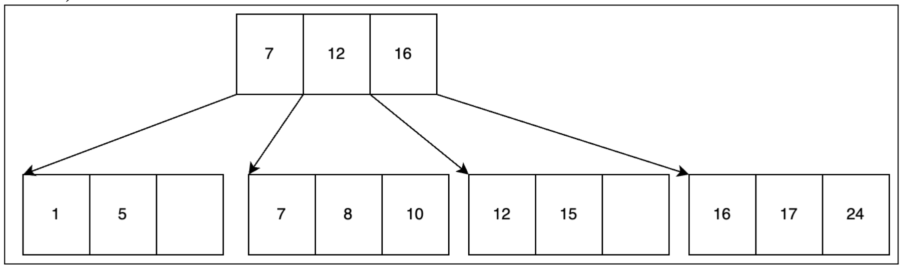
☐ A)



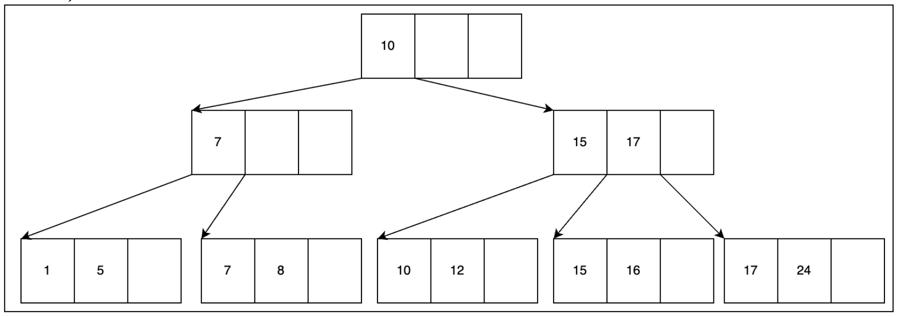
☐ B)



☐ C)



■ **D)**

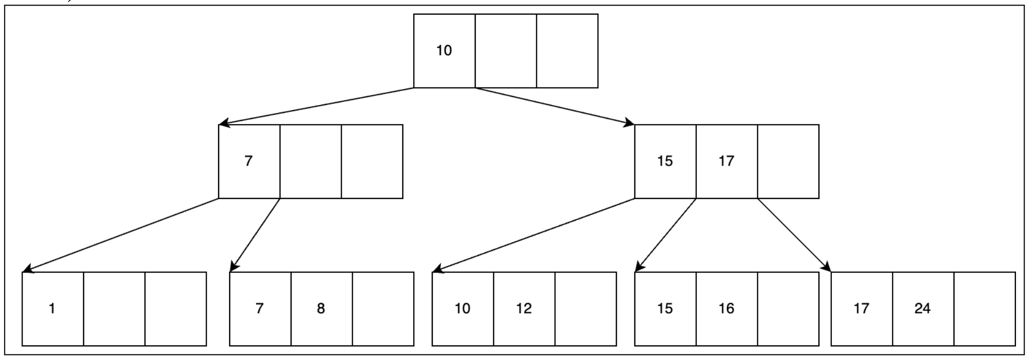


Solution: Inserting 16^* fills in the remaining space of the right-most leaf node. After inserting 16^*, inserting 17^* causes the right-most leaf node to split and as the root-level node is full, it causes recursive splits.

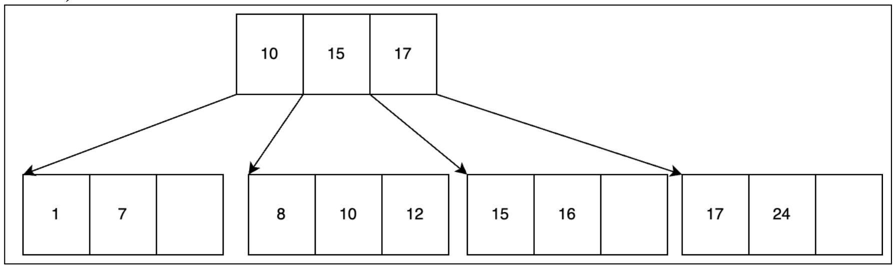
Question 4 continues. . .

(d) **[5 points]** Starting with the tree that results from **(c)**, deletes 5^*. Select the resulting tree.

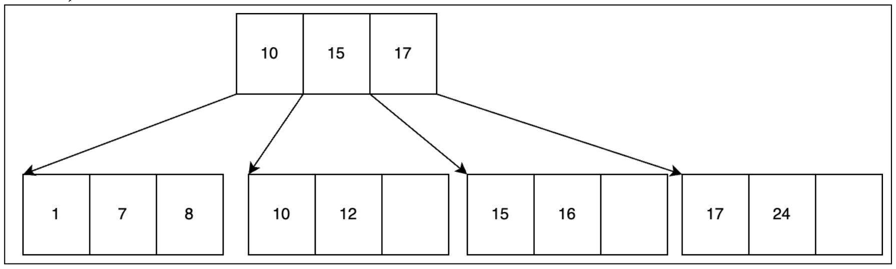
☐ A)



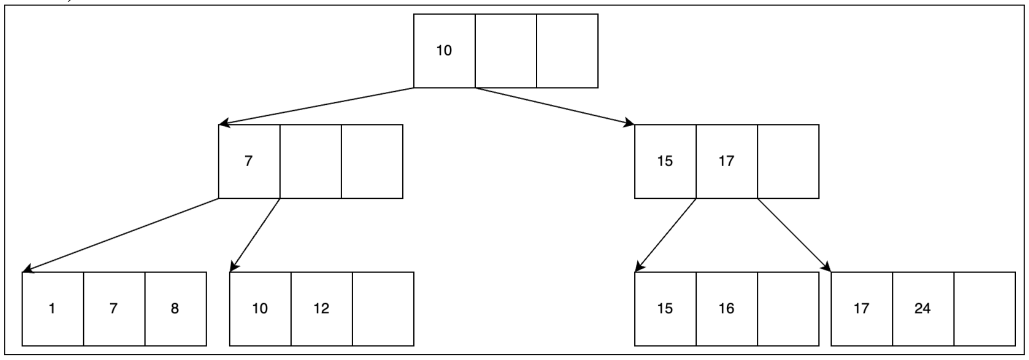
☐ B)



■ C)



☐ D)



Solution: Deleting 5^* causes the left-most leaf node underflow and merging of the first two leaf nodes. After merging, the number of pointers in the left internal node is less than 2. It causes recursive merging then.

Question 4 continues…

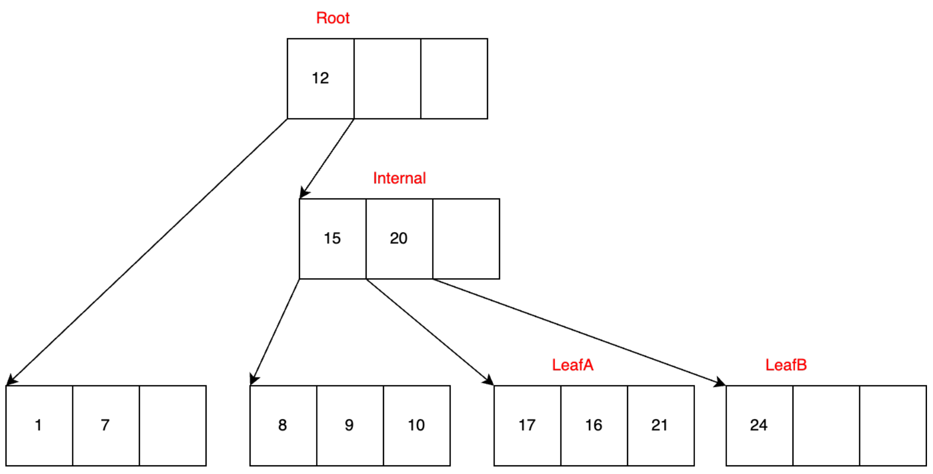


Figure 3: B+tree with violations

(e) The B+Tree shown in Figure 3 is invalid. That is, its nodes violate the correctness properties of B+Trees that we discussed in class. If the tree is invalid, select all the properties that are violated for each node. If the node is valid, then select 'None'. There will be **no** partial credit for missing violations.

Note:

- *If a node's subtrees are not the same height, the balance property is violated at that node only.*
- *If a node's subtrees contain values not in the range specified by the node's separator keys (e.g. internal node has separator keys as 5,10,18, then the first pointer pointed leaf node values should be less than 5, the second one should be in the range of [5, 10), etc. The logic also applies to any upper level internal node or root node. If the values in the leaf node satisfies its parent node's separator key property but violates the root node's, mark root node as separator key violation and not its parent node.), the separator keys property is violated at that node.*

i. **[2 points]** Which properties are violated by **Leaf A**?

■ **Key order property** ☐ Half-full property ☐ Balance property
☐ Separator keys ☐ None

Solution: Keys should be in the order of 17, 18, 21.

ii. **[2 points]** Which properties are violated by **LeafB**?

☐ Key order property ■ **Half-full property** ☐ Balance property
☐ Separator keys ☐ None

Solution: There must be at least 2 keys.

iii. **[2 points]** Which properties are violated by **Internal Node**?

☐ Key order property ☐ Half-full property ☐ Balance property
■ **Separator keys** ☐ None

Solution: The node's middle child contains 21. According to the node's separator keys, the middle child can only contain values between 15 (inclusive) and 20 (exclusive).

iv. **[2 points]** Which properties are violated by **Root**?
☐ Key order property ☐ Half-full property ■ **Balance property**
■ **Separator keys** ☐ None

Solution: The root is imbalanced, as it has both Leaf Node and Internal Node as its children. The root's subtree containing the second Leaf Node contains the value 8,9,10, violating the root's separator keys.

(f) i. **[2 points]** A read-only thread needs to hold at most one latches at the same time.
☐ True ■ **False**

Solution: A reader needs a latch on the parent and child (or two siblings in a leaf node scan) at most.

ii. **[2 points]** A write thread needs to hold at most two write latches at the same time
☐ True ■ **False**

Solution: A thread may need to hold an arbitrarily large number of write latches, for every node from the root to a leaf (plus potentially siblings of nodes along the path), in circumstances where the write might cause a rebalancing of the root

iii. **[2 points]** A write thread must release its latches in the order they were acquired (i.e., FIFO) to prevent concurrency errors.
☐ True ■ **False**

Solution: Threads can release latches in any order, although FIFO is best for performance.