Lecture #10: Sorting & Aggregation Algorithms

15-445/645 Database Systems (Spring 2024)
`https://15445.courses.cs.cmu.edu/spring2024/`
Carnegie Mellon University
Jignesh Patel

1 Query Plan

Up to this point we have talked about access methods. Now we need to actually execute the queries.

The database system will compile SQLs into query plans. The query plan is a tree of operators. We will cover this later in query execution lectures.

For a disk-oriented database system, we will use the buffer pool to implement algorithms that need to spill to disk. We want to minimize I/O for an algorithm.

2 Sorting

DBMSs need to sort data because tuples in a table have no specific order under the relation model. Sorting is (potentially) used in `ORDER BY`, `GROUP BY`, `JOIN`, and `DISTINCT` operators. If the data that needs to be sorted fits in memory, then the DBMS can use a standard sorting algorithm (e.g., quicksort). If the data does not fit, then the DBMS needs to use external sorting that is able to spill to disk as needed and prefers sequential over random I/O.

Top-N Heap Sort: If a query contains an `ORDER BY` with a `LIMIT`, then to find the top-N elements, the DBMS only needs to scan the data once while maintaining a priority queue of the top-N elements it has seen so far. The ideal scenario for heapsort is when the top-N elements fit in memory, so the DBMS can maintain the entire priority queue in-memory.

External merge sort: This is the standard algorithm for sorting data which is too large to fit in memory. It is a divide-and-conquer sorting algorithm that splits the data set into separate *runs* and then sorts them individually. It can spill runs to disk as needed then read them back in one at a time. The algorithm is comprised of two phases:

- **Phase #1 – Sorting:** First, the algorithm sorts small chunks of data that fit in main memory, and then writes the sorted pages back to disk.
- **Phase #2 – Merge:** Then, the algorithm combines the sorted runs into larger sorted runs.

A sorted run can be early-materialized, which means that the entire tuple is stored in the pages, or can be late-materialized, which means we only store record IDs in the pages and read them later.

Two-way Merge Sort

The most basic version of the external merge sort algorithm is the two-way merge sort. During the sorting phase, the algorithm reads each page, sorts it, and writes the sorted version back to disk. Then, in the merge phase, it uses three buffer pages. It reads two sorted pages in from disk, and merges them together into a third buffer page. Whenever the third page fills up, it is written back to disk and replaced with an empty page. Each set of sorted pages is called a *run*. The algorithm then recursively merges the runs together.

Let N be the total number of data pages. The algorithm makes $1 + \lceil log_2 N \rceil$ total passes through the data (1 for the first sorting step then $\lceil log_2 N \rceil$ for the recursive merging). The total I/O cost is $2N \times$ (# of passes) since each pass performs an I/O read and an I/O write for each page.

General (K-way) Merge Sort

The generalized version of the algorithm allows the DBMS to take advantage of using more than three buffer pages. Let B be the total number of buffer pages available. Then, during the sort phase, the algorithm can read and sort B pages at a time, so the DBMS writes $\lceil \frac{N}{B} \rceil$ sorted runs of length B pages back to disk. The merge phase can combine up to $B - 1$ runs in each pass, using one buffer page per run to read it and again using one buffer page for the combined data, writing back to disk as needed.

In the generalized version, the algorithm performs $1 + \lceil log_{B-1} \lceil \frac{N}{B} \rceil \rceil$ passes (one for the sorting phase and $\lceil log_{B-1} \lceil \frac{N}{B} \rceil \rceil$ for the merge phase. Then, the total I/O cost is $2N \times$ (# of passes) since it again has to make a read and write for each page in each pass.

Double Buffering Optimization

One optimization for external merge sort is prefetching the next run in the background and storing it in a second buffer while the system is processing the current run. This reduces the wait time for I/O requests at each step by continuously utilizing the disk. This optimization requires the use of multiple threads, since the prefetching should occur while the computation for the current run is happening.

Comparison Optimizations

Code specialization is often used to speedup sorting comparisions. Instead of providing the comparator as a function pointer to the sorting algorithm, the sorting function can be hard-coded to the specific key type. An example of this is template specialization in C++. Another optimization (for string based comparisons) is **suffix truncation**, wherein the binary prefix of long VARCHAR keys can be compared for equality checks with a fallback to the slower string comparison if the prefixes are equal.

Using B+Trees

It is sometimes advantageous for the DBMS to use an existing B+tree index to aid in sorting rather than using the external merge sort algorithm. In particular, if the index is a clustered index, the DBMS can just traverse the B+tree. Since the index is clustered, the data will be stored in the correct order, so the I/O access will be sequential. This means it is always better than external merge sort since no computation is required. On the other hand, if the index is unclustered, traversing the tree is almost always worse, since each record could be stored in any page, so nearly all record accesses will require a disk read.

3 Aggregations

An aggregation operator in a query plan collapses the values of one or more tuples into a single scalar value. There are two approaches for implementing an aggregation: (1) sorting and (2) hashing.

Sorting

The DBMS first sorts the tuples on the `GROUP BY` key(s). It can use either an in-memory sorting algorithm if everything fits in the buffer pool (e.g., quicksort) or the external merge sort algorithm if the size of the data exceeds memory. The DBMS then performs a sequential scan over the sorted data to compute the aggregation. The output of the operator will be sorted on the keys.

When performing sorting aggregations, it is important to order the query operations to maximize efficiency. For example, if the query requires a filter, it is better to perform the filter first and then sort the filtered data to reduce the amount of data that needs to be sorted.

Hashing

Hashing can be computationally cheaper than sorting for computing aggregations, especially when the order of the output does not matter. The DBMS populates an ephemeral hash table as it scans the table. For each record, check whether there is already an entry in the hash table and perform the appropriate modification. If the size of the hash table is too large to fit in memory, then the DBMS has to spill it to disk. There are two phases to accomplish this:

- **Phase #1 – Partition:** Use a hash function h_1 to split tuples into partitions on disk based on target hash key. This will put all tuples that match into the same partition. Assume B buffer pages in total. We will have B-1 output buffer pages for partitions and 1 buffer page for input data. If any partition is full, the DBMS will spill it to disk. Thus, this phase results in $B - 1$ partitions.
- **Phase #2 – ReHash:** For each partition on disk, read its pages into memory and build an in-memory hash table based on a second hash function h_2 (where $h_1 \neq h_2$). This will put all tuples that match into the same bucket. Then go through each bucket of this hash table to bring together matching tuples to compute the aggregation. This assumes that each partition fits in memory.

During the ReHash phase, the DBMS can store pairs of the form (`GroupByKey`→`RunningValue`) to compute the aggregation. The contents of `RunningValue` depends on the aggregation function (e.g. `(COUNT, SUM)` for `AVG`). To insert a new tuple into the hash table:

- If it finds a matching `GroupByKey`, then update the `RunningValue` appropriately.
- Else insert a new (`GroupByKey`→`RunningValue`) pair.

In general for aggregation, hashing is always more efficient unless the data is already sorted beforehand (e.g. following `Order By`).