Lecture #10

Sorting & Aggregations
ADMINISTRIVIA

Project #1 is due Feb 18\textsuperscript{th} @ 11:59pm
→ Special Office Hours: Feb 17\textsuperscript{th} @ 3:00-5:00pm GHC 4405

**Homework #2** is due Wed Feb 16\textsuperscript{th} @ 11:59pm

**Project #2** is due Wed March 13\textsuperscript{th} @ 11:59pm

**Mid-Term Exam** is on Wednesday Feb 28\textsuperscript{th}
→ During regular class time from 12:30-1:50pm
→ Please contact us if you need accommodations.
→ Review session on Feb 26 during regular class time. Not recorded!
  Bring your questions to class – it will be an interactive review session.
We are now going to talk about how to execute queries using the DBMS components we have discussed so far.

Next four lectures:
→ Operator Algorithms
→ Query Processing Models
→ Runtime Architectures
The operators are arranged in a tree.

Data flows from the leaves of the tree up towards the root.

→ We will discuss the granularity of the data movement next week.

The output of the root node is the result of the query.
DISK-ORIENTED DBMS

Just like it cannot assume that a table fits entirely in memory, a disk-oriented DBMS cannot assume that query results fit in memory.

We will use the buffer pool to implement algorithms that need to spill to disk.

We are also going to prefer algorithms that maximize the amount of sequential I/O.
WHY DO WE NEED TO SORT?

Relational model/SQL is unsorted.

Queries may request that tuples are sorted in a specific way (ORDER BY).

But even if a query does not specify an order, we may still want to sort to do other things:
→ Trivial to support duplicate elimination (DISTINCT).
→ Bulk loading sorted tuples into a B+Tree index is faster.
→ Aggregations (GROUP BY).
IN-MEMORY SORTING

Most **database systems** use Quicksort for in-memory sorting.

In other **data platforms**, notably Python – the default sort algorithm is TimSort. It is a combination of insertion and binary merge sort. Often works well on real data.

If data fits in memory, then we can use a standard sorting algorithm like Quicksort.

If data does not fit in memory, then we need to use a technique that is aware of the cost of reading and writing disk pages …
https://www.toptal.com/developers/sorting-algorithms

https://visualgo.net/en/sorting
TODAY'S AGENDA

Top-N Heap Sort
External Merge Sort
Aggregations
TOP-N HEAP SORT

If a query contains an **ORDER BY** with a **LIMIT**, then the DBMS only needs to scan the data once to find the top-N elements.

Ideal scenario for **heapsort**: if the top-N elements fit in memory.

→ Scan data once, maintain an in-memory sorted priority queue.

```
SELECT * FROM enrolled
ORDER BY sid
FETCH FIRST 4 ROWS
WITH TIES
```
TOP-N HEAP SORT

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**Original Data**

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3 4 6 2 9 1 4 4 8
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Original Data

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3 4 6 2 9 1 4 4 8
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Sorted Heap

```
3
```
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Sorted Heap

6 4 3

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SELECT * FROM enrolled
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**Original Data**

| 3 | 4 | 6 | 2 | 9 | 1 | 4 | 4 | 8 |

**Sorted Heap**

| 4 | 3 | 2 | 1 |
TOP-N HEAP SORT

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| 4 | 4 | 3 | 2 | 1 |
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SELECT * FROM enrolled
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**Original Data**

```
3 4 6 2 9 1 4 4 8
```

**Sorted Heap**

```
4 4 4 3 2 1
```

**Output**
EXTERNAL MERGE SORT

Divide-and-conquer algorithm that splits data into separate runs, sorts them individually, and then combines them into longer sorted runs.

Phase #1 – Sorting

→ Sort chunks of data that fit in memory and then write back the sorted chunks to a file on disk.

Phase #2 – Merging

→ Combine sorted runs into larger chunks.
SORTED RUN

A run is a list of key/value pairs.

**Key:** The attribute(s) to compare to compute the sort order.

**Value:** Two choices

→ Tuple *(early materialization).*
→ Record ID *(late materialization).*

**Early Materialization**

<table>
<thead>
<tr>
<th>K1</th>
<th>&lt;Tuple Data&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>K2</td>
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SORTED RUN

A run is a list of key/value pairs.

**Key:** The attribute(s) to compare to compute the sort order.

**Value:** Two choices
- Tuple (*early materialization*).
- Record ID (*late materialization*).
We will start with a simple example of a 2-way external merge sort.

→ “2” is the number of runs that we are going to merge into a new run for each pass.

Data is broken up into $N$ pages.

The DBMS has a finite number of $B$ buffer pool pages to hold input and output data.
SIMPLIFIED 2-WAY EXTERNAL MERGE SORT

Pass #0
→ Read one page of the table into memory
→ Sort page into a “run” and write it back to disk
→ Repeat until the whole table has been sorted into runs

Pass #1,2,3,...
→ Recursively merge pairs of runs into runs twice as long
→ Need at least 3 buffer pages (2 for input, 1 for output)
SIMPLIFIED 2-WAY EXTERNAL MERGE SORT

In each pass, we read and write every page in the file.

Number of passes
\[ = 1 + \lceil \log_2 N \rceil \]

Total I/O cost
\[ = 2N \cdot (\# \text{ of passes}) \]
SIMPLIFIED 2-WAY EXTERNAL MERGE SORT

In each pass, we read and write every page in the file.

Number of passes
= 1 + ⌈ log₂ N ⌉

Total I/O cost
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SIMPLIFIED 2-WAY EXTERNAL MERGE SORT

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Simplified 2-Way External Merge Sort

In each pass, we read and write every page in the file.

Number of passes

\[= 1 + \lceil \log_2 N \rceil\]

Total I/O cost

\[= 2N \cdot (\text{# of passes})\]
SIMPLIFIED 2-WAY EXTERNAL MERGE SORT

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SIMPLIFIED 2-WAY EXTERNAL MERGE SORT

In each pass, we read and write every page in the file.

Number of passes

= \(1 + \lceil \log_2 N \rceil\)

Total I/O cost

= \(2N \cdot (\# \text{ of passes})\)
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In each pass, we read and write every page in the file.

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Total I/O cost  
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In each pass, we read and write every page in the file.

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In each pass, we read and write every page in the file.

Number of passes
= \(1 + \lceil \log_2 N \rceil\)

Total I/O cost
= \(2N \cdot (\text{# of passes})\)
SIMPLIFIED 2-WAY EXTERNAL MERGE SORT

This algorithm only requires three buffer pool pages to perform the sorting ($B=3$).
→ Two input pages, one output page

But even if we have more buffer space available ($B>3$), it does not effectively utilize them if the worker must block on disk I/O...
GENERAL EXTERNAL MERGE SORT

Pass #0
→ Use $B$ buffer pages
→ Produce $\lceil N / B \rceil$ sorted runs of size $B$

Pass #1,2,3,...
→ Merge $B-1$ runs (i.e., K-way merge)

Number of passes = $1 + \lceil \log_{B-1} \lceil N / B \rceil \rceil$
Total I/O Cost = $2N \cdot (\# \text{ of passes}$)
EXAMPLE

Determine how many passes it takes to sort 108 pages with 5 buffer pool pages: \( N=108, B=5 \)

→ **Pass #0:** \( \lceil N / B \rceil = \lceil 108 / 5 \rceil = 22 \) sorted runs of 5 pages each
  (last run is only 3 pages).
→ **Pass #1:** \( \lceil N'/B-1 \rceil = \lceil 22 / 4 \rceil = 6 \) sorted runs of 20 pages each
  (last run is only 8 pages).
→ **Pass #2:** \( \lceil N''/B-1 \rceil = \lceil 6 / 4 \rceil = 2 \) sorted runs, first one has 80
  pages and second one has 28 pages.
→ **Pass #3:** Sorted file of 108 pages.

\[
1+\lceil \log_{B-1}[N / B] \rceil = 1+\lceil \log_4 22 \rceil = 1+\lceil 2.229... \rceil = 4 \text{ passes}
\]
DOUBLE BUFFERING OPTIMIZATION

Prefetch the next run in the background and store it in a second buffer while the system is processing the current run.

→ Reduces the wait time for I/O requests at each step by continuously utilizing the disk.
DOUBLE BUFFERING

Prefetch the next run in the background and store it in a second buffer while the system is processing the current run.

→ Overlap CPU and I/O operations
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Diagram:
- Disk
- Buffer Pool
- Page
- Output
- Merged page
- To Disk
- Sorted run
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Impact: reduces the effective “B” by half.

Reduces response time. What about throughput?
COMPARISON OPTIMIZATIONS

Approach #1: Code Specialization
→ Instead of providing a comparison function as a pointer to sorting algorithm, create a hardcoded version of sort that is specific to a key type.

Approach #2: Suffix Truncation
→ First compare a binary prefix of long VARCHAR keys instead of slower string comparison. Fallback to slower version if prefixes are equal.
USING B+TREES FOR SORTING

If the table that must be sorted already has a B+Tree index on the sort attribute(s), then we can use that to accelerate sorting.

Retrieve tuples in desired sort order by simply traversing the leaf pages of the tree.

Cases to consider:
→ Clustered B+Tree
→ Unclustered B+Tree
CASE #1 – CLUSTERED B+TREE

Traverse to the left-most leaf page, and then retrieve tuples from all leaf pages.

This is always better than external sorting because there is no computational cost, and all disk access is sequential.
CASE #2 – UNCLUSTERED B+TREE

Chase each pointer to the page that contains the data.

This is almost always a bad idea. In general, one I/O per data record.
AGGREGATIONS

Collapse values for a single attribute from multiple tuples into a single scalar value.

The DBMS needs a way to quickly find tuples with the same distinguishing attributes for grouping.

Two implementation choices:
→ Sorting
→ Hashing
### SORTING AGGREGATION

**enrolled (sid, cid, grade)**

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<th>cid</th>
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<tbody>
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**SELECT DISTINCT cid FROM enrolled WHERE grade IN ('B','C') ORDER BY cid**

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Filter | Remove Columns | Sort
SORTING AGGREGATION

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SELECT DISTINCT cid
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WHERE grade IN ('B', 'C')
ORDER BY cid
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Eliminate Duplicates
SORTING AGGREGATION

```
SELECT DISTINCT cid
FROM enrolled
WHERE grade IN ('B','C')
ORDER BY cid
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enrolled (sid, cid, grade)

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Filter

Remove Columns

Sort

Eliminate Duplicates
ALTERNATIVES TO SORTING

What if we do not need the data to be ordered?
→ Forming groups in GROUP BY (no ordering)
→ Removing duplicates in DISTINCT (no ordering)

Hashing is a better alternative in this scenario.
→ Only need to remove duplicates, no need for ordering.
→ Can be computationally cheaper than sorting.
HASHING AGGREGATE

Populate an ephemeral hash table as the DBMS scans the table. For each record, check whether there is already an entry in the hash table:

→ **DISTINCT**: Discard duplicate

→ **GROUP BY**: Perform aggregate computation

If everything fits in memory, then this is easy.

If the DBMS must spill data to disk, then we need to be smarter…
EXTERNAL HASHING AGGREGATE

Phase #1 – Partition
→ Divide tuples into buckets based on hash key
→ Write them out to disk when they get full

Phase #2 – ReHash
→ Build in-memory hash table for each partition and compute the aggregation
PHASE #1 – PARTITION

Use a hash function \( h_1 \) to split tuples into **partitions** on disk.

→ A partition is one or more pages that contain the set of keys with the same hash value.

→ Partitions are “spilled” to disk via output buffers.

Assume that we have \( B \) buffers.

We will use \( B-1 \) buffers for the partitions and 1 buffer for the input data.
**PHASE #1 – PARTITION**

**Filter**

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SELECT DISTINCT cid
FROM enrolled
WHERE grade IN ('B', 'C')
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<td>15-445</td>
<td>C</td>
</tr>
<tr>
<td>53688</td>
<td>15-826</td>
<td>B</td>
</tr>
<tr>
<td>53666</td>
<td>15-721</td>
<td>C</td>
</tr>
<tr>
<td>53655</td>
<td>15-445</td>
<td>C</td>
</tr>
</tbody>
</table>

**enrolled (sid, cid, grade)**

<table>
<thead>
<tr>
<th>sid</th>
<th>cid</th>
<th>grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>53666</td>
<td>15-445</td>
<td>C</td>
</tr>
<tr>
<td>53688</td>
<td>15-721</td>
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<td>15-721</td>
<td>C</td>
</tr>
<tr>
<td>53655</td>
<td>15-445</td>
<td>C</td>
</tr>
</tbody>
</table>

**B-1 partitions**

```
\( h_1 \)
```

```

15-445 15-445
15-445 15-445
15-312 15-445
15-826 15-210
15-721
```
PHASE #1 - PARTITION

SELECT DISTINCT cid
FROM enrolled
WHERE grade IN ('B', 'C')

enrolled (sid, cid, grade)

<table>
<thead>
<tr>
<th>sid</th>
<th>cid</th>
<th>grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>53666</td>
<td>15-445</td>
<td>C</td>
</tr>
<tr>
<td>53688</td>
<td>15-721</td>
<td>A</td>
</tr>
<tr>
<td>53688</td>
<td>15-826</td>
<td>B</td>
</tr>
<tr>
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<td>15-721</td>
<td>C</td>
</tr>
<tr>
<td>53655</td>
<td>15-445</td>
<td>C</td>
</tr>
</tbody>
</table>

Filter

<table>
<thead>
<tr>
<th>sid</th>
<th>cid</th>
<th>grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>53666</td>
<td>15-445</td>
<td>C</td>
</tr>
<tr>
<td>53688</td>
<td>15-721</td>
<td>C</td>
</tr>
<tr>
<td>53655</td>
<td>15-445</td>
<td>C</td>
</tr>
</tbody>
</table>

Remove Columns

B-1 partitions

\( h_1 \)

\[
\begin{align*}
&15-445 & 15-445 \\
&15-445 & 15-312 \\
&15-445 & 15-210 \\
&15-721 & 15-312 \\
&15-721 & 15-210 \\
\end{align*}
\]
PHASE #1 – PARTITION

**SELECT DISTINCT cid**
**FROM enrolled**
**WHERE grade IN ('B', 'C')**

<table>
<thead>
<tr>
<th>sid</th>
<th>cid</th>
<th>grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>53666</td>
<td>15-445</td>
<td>C</td>
</tr>
<tr>
<td>53688</td>
<td>15-826</td>
<td>B</td>
</tr>
<tr>
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<td>15-721</td>
<td>C</td>
</tr>
<tr>
<td>53655</td>
<td>15-445</td>
<td>C</td>
</tr>
</tbody>
</table>

**enrolled (sid, cid, grade)**

<table>
<thead>
<tr>
<th>sid</th>
<th>cid</th>
<th>grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>53666</td>
<td>15-445</td>
<td>C</td>
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<td>C</td>
</tr>
<tr>
<td>53655</td>
<td>15-445</td>
<td>C</td>
</tr>
</tbody>
</table>

**Filter**

**Remove Columns**

**B-1 partitions**
PHASE #2 – REHASH

For each partition on disk:
→ Read it into memory and build an in-memory hash table based on a second hash function $h_2$.
→ Then go through each bucket of this hash table to bring together matching tuples.

This assumes that each partition fits in memory.
**PHASE #2 - REHASH**

```sql
SELECT DISTINCT cid
FROM enrolled
WHERE grade IN ('B', 'C')
```

**enrolled (sid, cid, grade)**

<table>
<thead>
<tr>
<th>sid</th>
<th>cid</th>
<th>grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>53666</td>
<td>15-445</td>
<td>C</td>
</tr>
<tr>
<td>53688</td>
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<td>C</td>
</tr>
<tr>
<td>53655</td>
<td>15-445</td>
<td>C</td>
</tr>
</tbody>
</table>

**Phase #1 Buckets**

```
<table>
<thead>
<tr>
<th>Partitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-445</td>
</tr>
<tr>
<td>15-445</td>
</tr>
<tr>
<td>15-445</td>
</tr>
<tr>
<td>15-445</td>
</tr>
<tr>
<td>15-826</td>
</tr>
<tr>
<td>15-826</td>
</tr>
</tbody>
</table>
```

**PHASE #2 - REHASH**

```
SELECT DISTINCT cid
FROM enrolled
WHERE grade IN ('B', 'C')
```

**Phase #1 Buckets**

```
Phase #1 Buckets
```

**Hash Table**

```
<table>
<thead>
<tr>
<th>sid</th>
<th>cid</th>
<th>grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>53666</td>
<td>15-445</td>
<td>C</td>
</tr>
<tr>
<td>53688</td>
<td>15-721</td>
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<td>C</td>
</tr>
<tr>
<td>53655</td>
<td>15-445</td>
<td>C</td>
</tr>
</tbody>
</table>
```

**enrolled (sid, cid, grade)**
PHASE #2 – REHASH

```sql
SELECT DISTINCT cid
FROM enrolled
WHERE grade IN ('B', 'C')
```

**Phase #1 Buckets**

```
15-445 15-445
15-445 15-445
15-445 15-445
15-826 15-826
```

**Hash Table**

```
15-445
15-826
```

**enrolled (sid, cid, grade)**

<table>
<thead>
<tr>
<th>sid</th>
<th>cid</th>
<th>grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>53666</td>
<td>15-445</td>
<td>C</td>
</tr>
<tr>
<td>53688</td>
<td>15-721</td>
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<td>B</td>
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<tr>
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<td>15-721</td>
<td>C</td>
</tr>
<tr>
<td>53655</td>
<td>15-445</td>
<td>C</td>
</tr>
</tbody>
</table>
**PHASE #2 - REHASH**

**SELECT** DISTINCT cid 
**FROM** enrolled 
**WHERE** grade IN ('B', 'C')

**Phase #1 Buckets**

<table>
<thead>
<tr>
<th>Hash Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-445</td>
</tr>
<tr>
<td>15-826</td>
</tr>
</tbody>
</table>

**enrolled (sid, cid, grade)**

<table>
<thead>
<tr>
<th>sid</th>
<th>cid</th>
<th>grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>53666</td>
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<td>C</td>
</tr>
<tr>
<td>53688</td>
<td>15-721</td>
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<tr>
<td>53666</td>
<td>15-721</td>
<td>C</td>
</tr>
<tr>
<td>53655</td>
<td>15-445</td>
<td>C</td>
</tr>
</tbody>
</table>

**Final Result**

<table>
<thead>
<tr>
<th>cid</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-445</td>
</tr>
<tr>
<td>15-826</td>
</tr>
</tbody>
</table>
**PHASE #2 - REHASH**

**SELECT** DISTINCT cid
**FROM** enrolled
**WHERE** grade IN ('B','C')

**Phase #1 Buckets**

**enrolled (sid, cid, grade)**

<table>
<thead>
<tr>
<th>sid</th>
<th>cid</th>
<th>grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>53666</td>
<td>15-445</td>
<td>C</td>
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<tr>
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<td>15-721</td>
<td>C</td>
</tr>
<tr>
<td>53655</td>
<td>15-445</td>
<td>C</td>
</tr>
</tbody>
</table>

**Final Result**

<table>
<thead>
<tr>
<th>cid</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-445</td>
</tr>
<tr>
<td>15-826</td>
</tr>
</tbody>
</table>
**PHASE #2 – REHASH**

**SELECT** DISTINCT cid  
**FROM** enrolled  
**WHERE** grade IN ('B', 'C')

**Phase #1 Buckets**

**enrolled (sid, cid, grade)**

<table>
<thead>
<tr>
<th>sid</th>
<th>cid</th>
<th>grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>53666</td>
<td>15-445</td>
<td>C</td>
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<tr>
<td>53688</td>
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<tr>
<td>53666</td>
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<td>C</td>
</tr>
<tr>
<td>53655</td>
<td>15-445</td>
<td>C</td>
</tr>
</tbody>
</table>

**Final Result**

**Hash Table**

<table>
<thead>
<tr>
<th>cid</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-445</td>
</tr>
<tr>
<td>15-826</td>
</tr>
<tr>
<td>15-721</td>
</tr>
</tbody>
</table>
HASHING SUMMARIZATION

During the rehash phase, store pairs of the form \((\text{GroupKey} \rightarrow \text{RunningVal})\)

When we want to insert a new tuple into the hash table:
→ If we find a matching \text{GroupKey}, just update the \text{RunningVal} appropriately
→ Else insert a new \text{GroupKey} \rightarrow \text{RunningVal}
HASHING SUMMARIZATION

```sql
SELECT cid, AVG(s.gpa)
FROM student AS s, enrolled AS e
WHERE s.sid = e.sid
GROUP BY cid
```

### Phase #1

<table>
<thead>
<tr>
<th>Buckets</th>
<th>Hash Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-445 15-445</td>
<td>h2</td>
<td>(2, 7.32)</td>
</tr>
<tr>
<td>15-826</td>
<td>h2</td>
<td>(1, 3.33)</td>
</tr>
<tr>
<td>15-721</td>
<td>h2</td>
<td>(1, 2.89)</td>
</tr>
</tbody>
</table>

**Hash Table**

<table>
<thead>
<tr>
<th>key</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-445</td>
<td>(2, 7.32)</td>
</tr>
<tr>
<td>15-826</td>
<td>(1, 3.33)</td>
</tr>
<tr>
<td>15-721</td>
<td>(1, 2.89)</td>
</tr>
</tbody>
</table>
SELECT cid, AVG(s.gpa) 
FROM student AS s, enrolled AS e 
WHERE s.sid = e.sid 
GROUP BY cid

### Hash Table

<table>
<thead>
<tr>
<th>key</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-445</td>
<td>(2, 7.32)</td>
</tr>
<tr>
<td>15-826</td>
<td>(1, 3.33)</td>
</tr>
<tr>
<td>15-721</td>
<td>(1, 2.89)</td>
</tr>
</tbody>
</table>
HASHING SUMMARIZATION

SELECT cid, AVG(s.gpa)
FROM student AS s, enrolled AS e
WHERE s.sid = e.sid
GROUP BY cid

---

**Phase #1**

<table>
<thead>
<tr>
<th>Buckets</th>
<th>Hash Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-445</td>
<td>h_2</td>
</tr>
<tr>
<td>15-445</td>
<td>h_2</td>
</tr>
<tr>
<td>15-826</td>
<td>h_2</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>15-721</td>
<td>h_2</td>
</tr>
</tbody>
</table>

**Hash Table**

<table>
<thead>
<tr>
<th>key</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-445</td>
<td>(2, 7.32)</td>
</tr>
<tr>
<td>15-826</td>
<td>(1, 3.33)</td>
</tr>
<tr>
<td>15-721</td>
<td>(1, 2.89)</td>
</tr>
</tbody>
</table>

**Running Totals**

- AVG(col)  →  (COUNT, SUM)
- MIN(col)  →  (MIN)
- MAX(col)  →  (MAX)
- SUM(col)  →  (SUM)
- COUNT(col) →  (COUNT)
##### HASHING SUMMARIZATION

Select `cid, AVG(s.gpa)` from `student AS s, enrolled AS e` where `s.sid = e.sid` group by `cid`.

---

### Running Totals

- `AVG(col) -> (COUNT, SUM)`
- `MIN(col) -> (MIN)`
- `MAX(col) -> (MAX)`
- `SUM(col) -> (SUM)`
- `COUNT(col) -> (COUNT)`

---

### Hash Table

**Phase #1 Buckets**

- 15-445
- 15-445
- 15-826
- 15-826
- ...
- 15-721

**Hash Table**

<table>
<thead>
<tr>
<th>key</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-445</td>
<td>(2, 7.32)</td>
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<td>(1, 3.33)</td>
</tr>
<tr>
<td>15-721</td>
<td>(1, 2.89)</td>
</tr>
</tbody>
</table>

**Final Result**

<table>
<thead>
<tr>
<th>cid</th>
<th>AVG(gpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-445</td>
<td>3.66</td>
</tr>
<tr>
<td>15-826</td>
<td>3.33</td>
</tr>
<tr>
<td>15-721</td>
<td>2.89</td>
</tr>
</tbody>
</table>
CONCLUSION

Choice of sorting vs. hashing is subtle and depends on optimizations done in each case.

We already discussed the optimizations for sorting:
→ Chunk I/O into large blocks to amortize costs
→ Double-buffering to overlap CPU and I/O
NEXT CLASS

Nested Loop Join
Sort-Merge Join
Hash Join