Lecture #09: Indexes II

15-445/645 Database Systems (Spring 2025)

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Carnegie Mellon University
Jignesh Patel

1 Bloom filter

A **filter** is a data structure that answers set membership queries (does this element exist in the set?). If we know an element is not in a set, we save time finding it in the set while it does not exist. For example, within a chained hash table, we can put a filter at each bucket pointer. If the filter says negative, we then know the key is not in the chain thus saving our time traversing through the whole chain, which is costly.

A **Bloom filter** is a probabilistic filter implemented with bitmap. By probabilistic, it means a Bloom filter does not always give the correct answer to a set membership query (false positives). However, a Bloom filter guarantees that it will never has false negatives.

This implies that false positives could happen. The false positive rate can be calculated via Bloom Filter Calculator.

A Bloom filter needs to define

- Size of the bitmap
- Numbers of hash functions to use

Insert(x)

For insertion, the pre-defined hash functions are used on the inserted element x. For each function's output hash value, we modular it with the bitmap size, then set the corresponding position in the bitmap to one. See Figure 1.

Lookup(x)

For lookup, a similar operation is done on element x. Each hash function takes x as input and modular the output value with bitmap size. If any of the corresponding positions in the bitmap is not one, a false is returned. Otherwise a true is returned (one has to go to the set and see if it's actually in it).

Other Variations

- **Counting Bloom filter**: Instead of bits, a sequence of integers is used. Deletion is possible.
- **Cuckoo filter**: Same idea to Cuckoo Hash, but store fingerprints of elements instead. Deletion is also possible.
- **Succinct range filter**: An immutable filter. While no insertion can be made, a lookup can ask if there is an element within a range.

2 Skip List

A *Skip List* it uses multiple levels of linked lists to skip some nodes and thus traverse faster. See Figure 2.

Like the B+tree, it stores keys in an ordered manner. However, it does not require rebalancing during insertion or deletion. It is commonly seen in an in-memory data structure such as memtable.

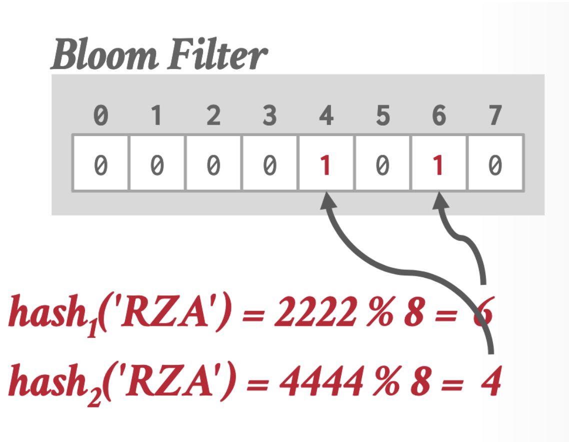

Figure 1: Insert 'RZA' into a Bloom filter with two hash functions

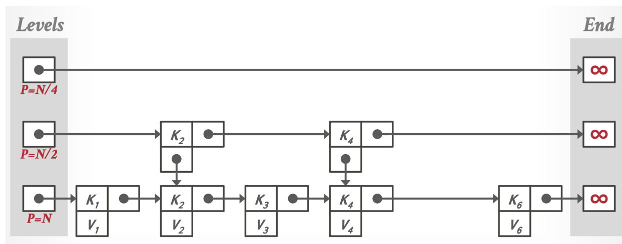

Figure 2: An overview of a skip list

Find

Go to the top-level linked list and traverse until the value is about to be greater than the target. Then go down to the next level and traverse the same way until it reaches the bottom list and then the target key.

Insert

Coins are flipped to decide until which level this new node is going to insert. Insertions on different levels are done from bottom to top, in order to keep the whole data structure intact. Otherwise, a reader from another thread may come across a node and find out there is no pointer to the lower level while the insertion is still not finished.

Note that if the linked list is in one direction, each level's insertion could be done by an atomic pointer in-memory swap (swap K_4's next with K_5's next pointer), thus no latch is needed.

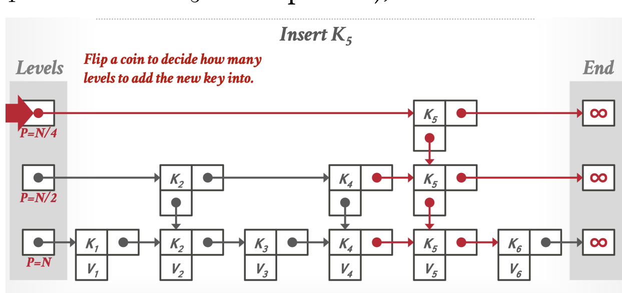

Figure 3: The skip list in Figure 2 after K_5 inserted. And it succeeded in two coin flips.

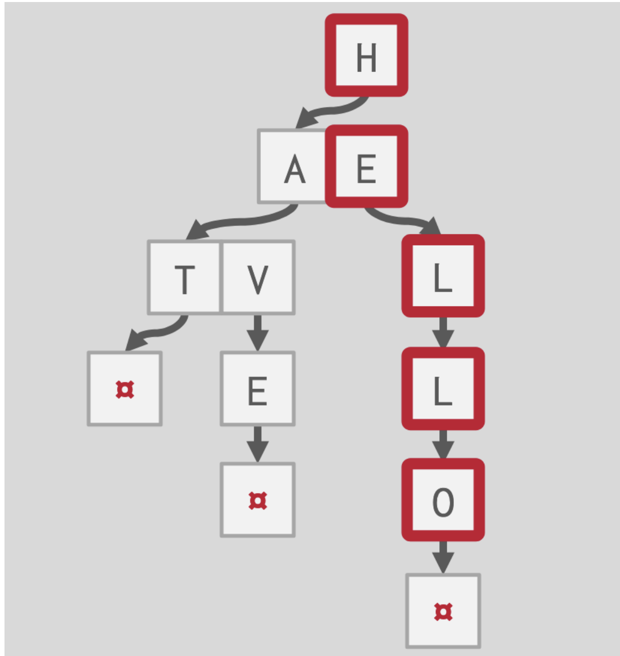

Figure 4: Finding a key "HELLO" in a trie

Delete

At first, a node is marked as deleted instead of removed from the data structure to prevent other reader threads from visiting the dead object. Any reader can ignore the deleted node and keep traversing the way down. Later when the node object is to be deleted, the top node will be removed before the bottom one to keep the data structure intact.

Conclusion

Advantages:

- Less memory usage if not including reverse pointer compared to B+tree.
- No rebalancing is needed while inserting and deleting.

Disadvantages:

- Not disk/cache friendly because they do not optimize locality of reference
- Reverse search is non-trivial, it becomes tricky to handle both ascending and descending scans.

3 Trie

Because a B+tree does not provide information about whether a node exists below an inner node or not, it's essential to go down to the leaf node to find out a node does not exist. It costs one buffer pool page miss per tree level.

A *trie* is an order-preserving data structure that stores keys as digits. A trivial way is to make characters (a byte) as digits for strings or bits for other data types. These digits form a tree structure to represent prefixes of every entry inserted into this trie. Note that the operation complexity depends on the length of the target key.

The span of each trie level is the number of bits that a digit takes. If the digit and its prefix exist in the corpus, then a pointer to the next level node is stored at that digit, otherwise, a null is stored.

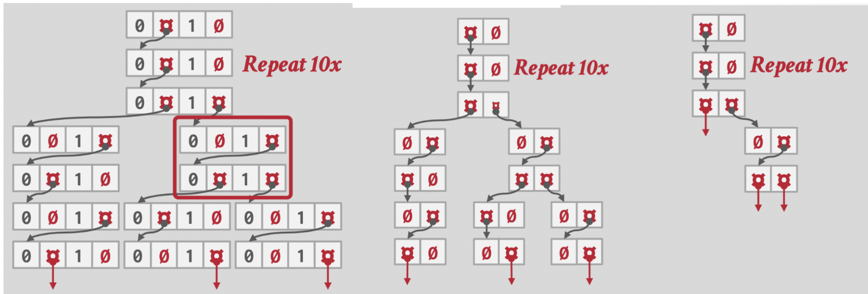

Figure 5: From left to right, horizontal compression and vertical compression are done on a trie with a one-bit span node

Compression

- If we have a known span of a level, we can horizontally compress the node to an array instead of a map. See Figure 5
- If a node has only a single child, we can vertically compress the nodes below the node as there are no branches down there. See Figure 5. This is also called Radix tree. False positives may happen, readers have to check the tuple a node points to.

4 Inverted Index

The indexes we talked about before are only good for point or range searches. They do not support keyword search. For example, a query to find all Wikipedia articles that contain the word "Pavlo".

An inverted index stores an immutable mapping of terms to records (the list of records is called a posting list) that contain those terms in the target attribute.

Lucene Implementation

Lucene is a specialized inverted index engine. The way it stores an inverted index is to have a trie-like data structure called a "finite state transducer" (See Figure 6). Instead of storing pointers to a tuple like a trie, it stores weights on every edge. By traversing down to the key one is looking for, a rolling sum of the weights will eventually give the exact position the entry is stored in the mapping.

In the dictionary of terms, since it's immutable and is built ahead of time, compression techniques such as delta compression, and bit-packing. Pre-aggregation is also supported to accelerate the query time of aggregation queries that group on terms.

PostgreSQL Implementation

PostgreSQL's *Generalized Inverted Index* uses a B+tree to build the term dictionary. The value of this B+tree leaf node depends on the size of the posting list. For small-sized posting lists, it will be a sorted list of record IDs. For large-sized posting lists, additional B+tree structures will be built to hold the record IDs.

A separate pending list is used to avoid small incremental updates. It logs updates and accumulates to a bulk insert to the dictionary.

5 Vector Index

Inverted indexes can support keyword search, but not the semantic meaning of the content (i.e., the keyword has to be contained in the content exactly). Suppose an application wants to search for records that

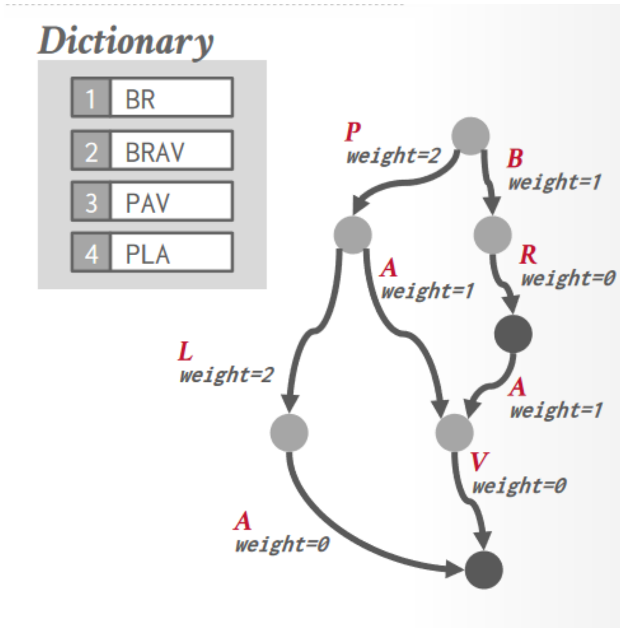

Figure 6: Finite state transducer to find the offset in the dictionary of a term

are **related** to a certain topic, for example, *"hip-hop groups with songs about slinging"*. In that case, we need another method other than the inverted index.

Large language models are known for generating **embeddings** for texts, an array of floating point numbers. Embeddings are geometrically close to each other if they have similar semantic meanings, therefore a vector index specialized for nearest-neighbor search can help in the query we were discussing.

However, there is no correct answer to this kind of query compared to those traditional queries we have seen before. There is also a need to filter data before or after finding nearest neighbors.

Inverted File

Partition vectors into smaller groups by a clustering algorithm. To search the nearest neighbors, use the same clustering algorithm to locate the query to a group, then look up all the vectors within that group (might also want to look at the groups nearby). Example: IVFFlat.

Preprocessing and quantization can be performed on the index to reduce the dimension and thus speed up the lookup time, while the original vectors are still preserved at their original locations.

Navigable Small Worlds

Build a graph that represents the neighbor relationship between vectors, where each node is a vector and its edges link to its n nearest neighbors. Navigable small worlds use this graph to navigate from the entry point to the query vector by greedily choosing the edge that moves closer to the query vector. Example: FAISS, HNSWlib