

Carnegie Mellon University

Database Systems Join Algorithms



15-445/645 SPRING 2025



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ADMINISTRIVIA

Mid-term Exam on Wednesday Feb 26th (next class)

→ In-class; in this room.

→ Study guide is available [online](#). What to bring to the exam?

- Your CMU ID (Mandatory)
- A calculator is recommended (e.g., logarithms)
- A single letter-size page of handwritten notes. You may use both sides.

UPCOMING DATABASE TALKS

DB Seminar

Apache Pinot Query Optimizer

→ Monday Feb 24th @ 4:30pm ET

→ <https://cmu.zoom.us/j/93441451665>

LAST CLASS

We started discussing how to implement algorithms to compute queries and handle data sets that are larger than available memory.

→ **Common Pattern:** Divide-and-Conquer

There are two high-level strategies to quickly find tuples with the same attribute values.

→ Sorting

→ Hashing

WHY DO WE NEED TO JOIN?

We normalize tables in a relational database to avoid unnecessary repetition of information.

We then use the join operator to reconstruct the original tuples without any information loss.

JOIN ALGORITHMS

We will focus on performing binary joins (two tables) using **inner equijoin** algorithms.

- These algorithms can be tweaked to support other joins.
- Multi-way joins exist primarily in research literature (e.g., **worst-case optimal joins**).

In general, we want the smaller table to always be the left table ("outer table") in the query plan.

- The optimizer will (try to) figure this out when generating the physical plan.

QUERY PLAN

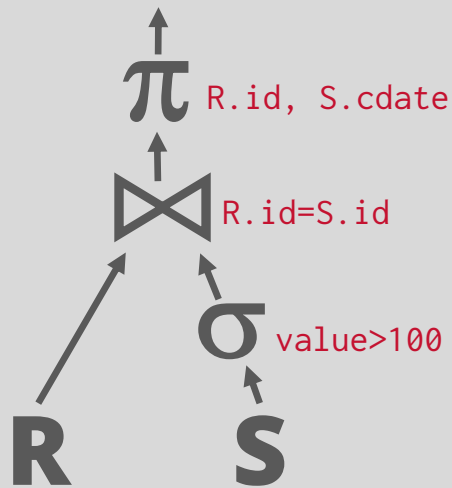
The operators are arranged in a tree.

Data flows from the leaves of the tree up towards the root.

→ We will discuss the granularity of the data movement next lecture.

The output of the root node is the result of the query.

```
SELECT R.id, S.cdate
FROM R JOIN S
ON R.id = S.id
WHERE S.value > 100
```



JOIN OPERATORS

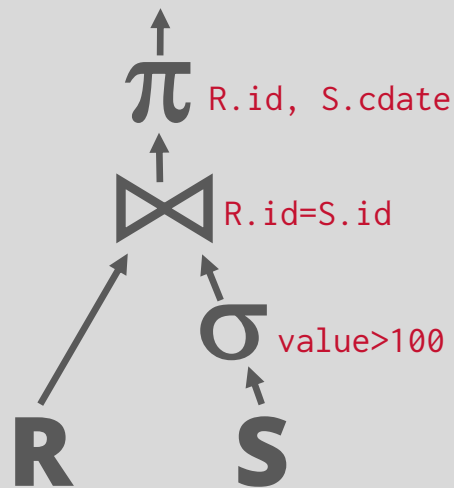
Decision #1: Output

→ What data does the join operator emit to its parent operator in the query plan tree?

Decision #2: Cost Analysis Criteria

→ How do we determine whether one join algorithm is better than another?

```
SELECT R.id, S.cdate
FROM R JOIN S
ON R.id = S.id
WHERE S.value > 100
```



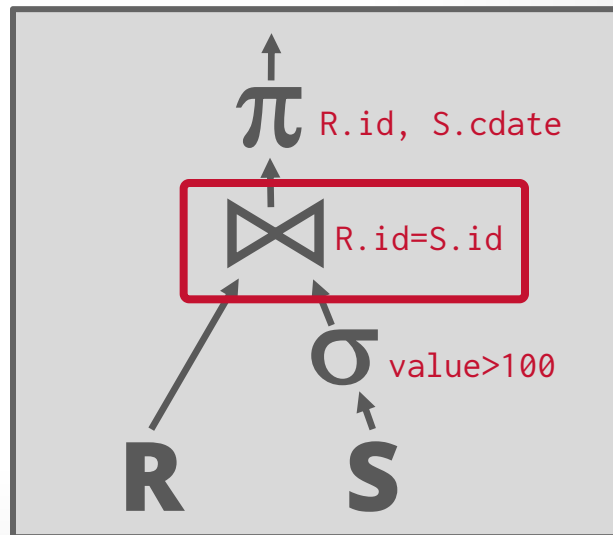
OPERATOR OUTPUT

For tuple $\mathbf{r} \in \mathbf{R}$ and tuple $\mathbf{s} \in \mathbf{S}$ that match on join attributes, concatenate \mathbf{r} and \mathbf{s} together into a new tuple.

Output contents can vary:

- Depends on processing model
- Depends on storage model
- Depends on data requirements in query

```
SELECT R.id, S.cdate
FROM R JOIN S
ON R.id = S.id
WHERE S.value > 100
```



OPERATOR OUTPUT: DATA

Early Materialization:

→ Copy the values for the attributes in outer and inner tuples into a new output tuple.

```
SELECT R.id, S.cdate
FROM R JOIN S
ON R.id = S.id
WHERE S.value > 100
```

OPERATOR OUTPUT: DATA

Early Materialization:

→ Copy the values for the attributes in outer and inner tuples into a new output tuple.

```
SELECT R.id, S.cdate
FROM R JOIN S
ON R.id = S.id
WHERE S.value > 100
```

R(id, name)

id	name
123	abc

S(id, value, cdate)

id	value	cdate
123	1000	2/24/25
123	2000	2/24/25

OPERATOR OUTPUT: DATA

Early Materialization:

→ Copy the values for the attributes in outer and inner tuples into a new output tuple.

```
SELECT R.id, S.cdate
FROM R JOIN S
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WHERE S.value > 100
```

R(id, name) **S(id, value, cdate)**

id	name		id	value	cdat
123	abc	⋈	123	1000	2/24/25
			123	2000	2/24/25


R.id	R.name	S.id	S.value	S.cdate
123	abc	123	1000	2/24/25
123	abc	123	2000	2/24/25

OPERATOR OUTPUT: DATA

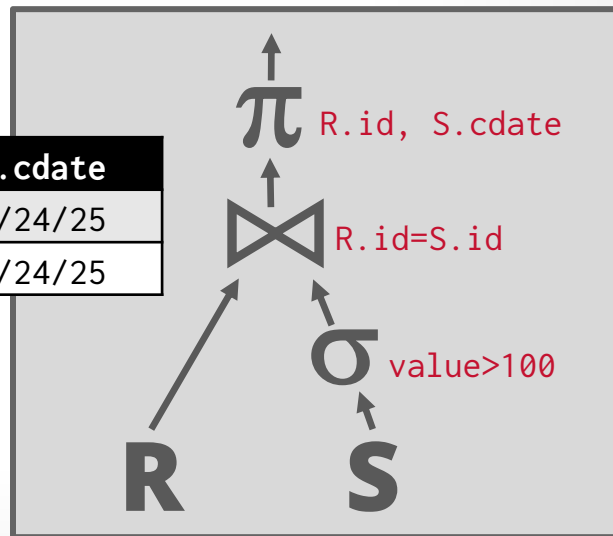
Early Materialization:

→ Copy the values for the attributes in outer and inner tuples into a new output tuple.

```
SELECT R.id, S.cdate
FROM R JOIN S
ON R.id = S.id
WHERE S.value > 100
```



R.id	R.name	S.id	S.value	S.cdate
123	abc	123	1000	2/24/25
123	abc	123	2000	2/24/25



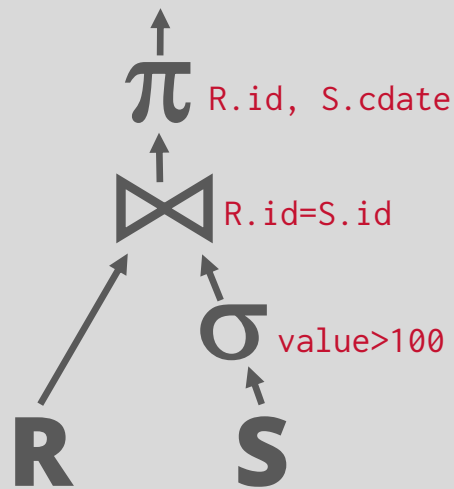
OPERATOR OUTPUT: DATA

Early Materialization:

→ Copy the values for the attributes in outer and inner tuples into a new output tuple.

Subsequent operators in the query plan never need to go back to the base tables to get more data.

```
SELECT R.id, S.cdate
FROM R JOIN S
ON R.id = S.id
WHERE S.value > 100
```



OPERATOR OUTPUT: RECORD IDS

Late Materialization:

→ Only copy the joins keys along with the Record IDs of the matching tuples.

```
SELECT R.id, S.cdate
FROM R JOIN S
ON R.id = S.id
WHERE S.value > 100
```

R(id, name)

id	name
123	abc

S(id, value, cdate)

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OPERATOR OUTPUT: RECORD IDS

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R(id, name) **S(id, value, cdate)**

id	name		id	value	cdate
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			123	2000	2/24/25

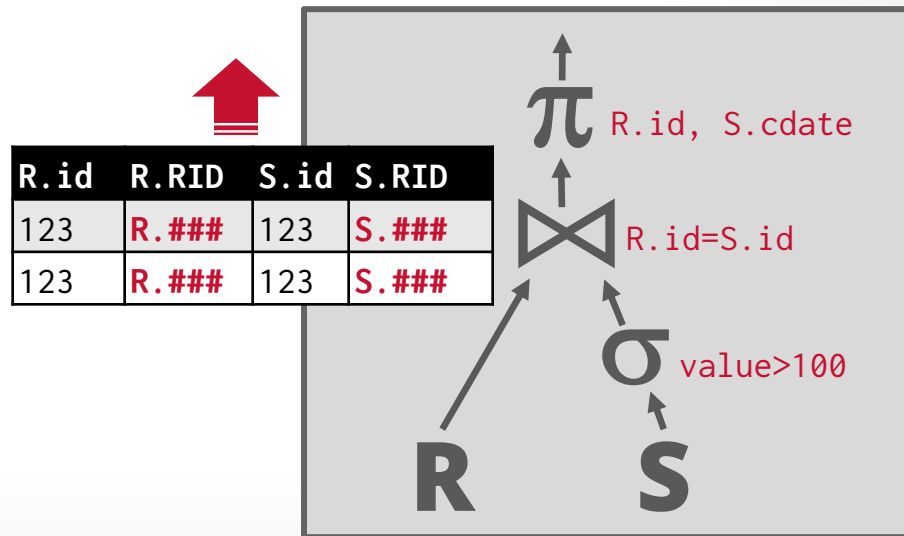
R.id	R.RID	S.id	S.RID
123	R.###	123	S.###
123	R.###	123	S.###

OPERATOR OUTPUT: RECORD IDS

Late Materialization:

→ Only copy the joins keys along with the Record IDs of the matching tuples.

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SELECT R.id, S.cdate
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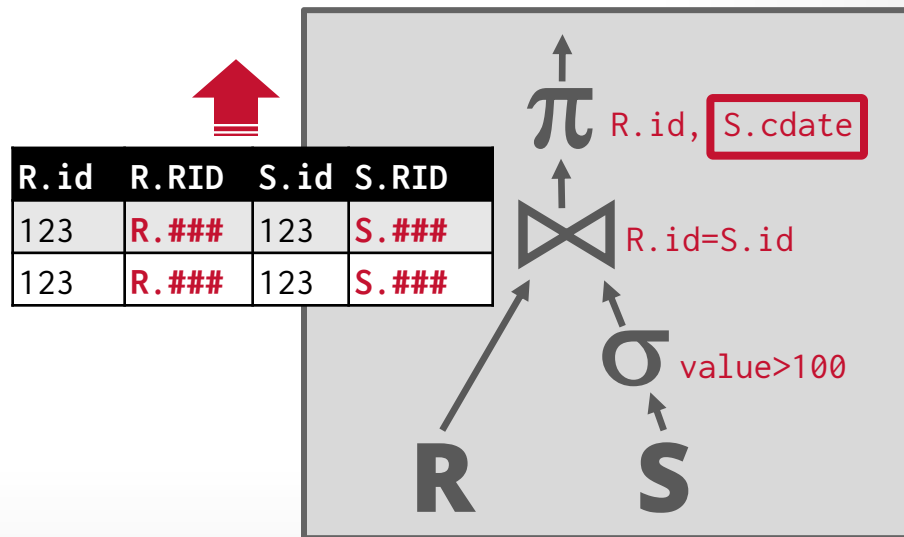


OPERATOR OUTPUT: RECORD IDS

Late Materialization:

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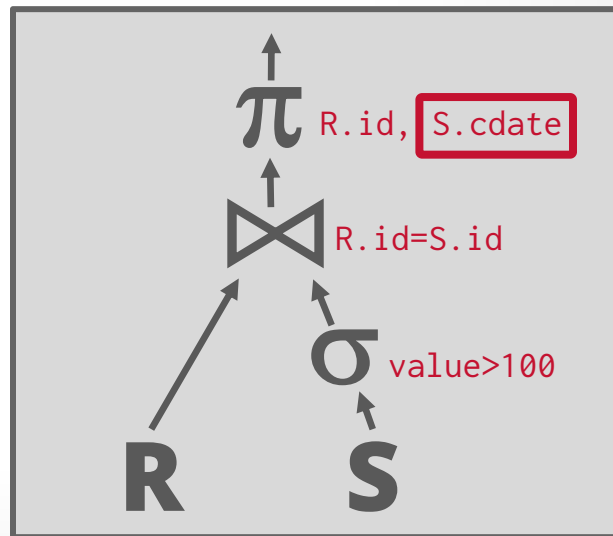
OPERATOR OUTPUT: RECORD IDS

Late Materialization:

→ Only copy the joins keys along with the Record IDs of the matching tuples.

Ideal for column stores because the DBMS does not copy data that is not needed for the query.

```
SELECT R.id, S.cdate
FROM R JOIN S
ON R.id = S.id
WHERE S.value > 100
```



COST ANALYSIS CRITERIA

Given a query that joins table **R** with table **S**, assume the DBMS has the following information those tables:

→ **M** pages in table **R**, **m** tuples in **R**

→ **N** pages in table **S**, **n** tuples in **S**

```
SELECT R.id, S.cdate
FROM R JOIN S
      ON R.id = S.id
WHERE S.value > 100
```

Cost Metric: # of I/Os to compute join

→ Ignore result output costs because it depends on the data and is the same for all algorithms.

→ Ignore computation / network costs (for now).

JOIN ALGORITHMS

Nested Loop Join

- Naïve
- Block
- Index

Sort-Merge Join

Hash Join

- Simple
- GRACE (Externally Partitioned)
- Hybrid

NAÏVE NESTED LOOP JOIN

```
foreach tuple  $r \in R$ :  
  foreach tuple  $s \in S$ :  
    if  $r$  and  $s$  match then emit
```

R(id, name)

id	name
600	MethodMan
200	GZA
100	Andy
300	ODB
500	RZA
700	Ghostface
400	Raekwon

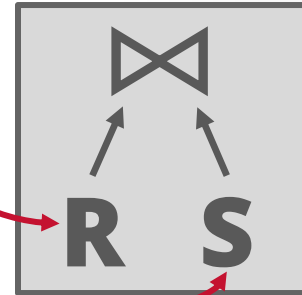
S(id, value, cdate)

id	value	cdate
100	2222	2/24/25
500	7777	2/24/25
400	6666	2/24/25
100	9999	2/24/25
200	8888	2/24/25

NAÏVE NESTED LOOP JOIN

```

foreach tuple  $r \in R$ : ← Outer
  foreach tuple  $s \in S$ : ← Inner
    if  $r$  and  $s$  match then emit
  
```



R(id, name)

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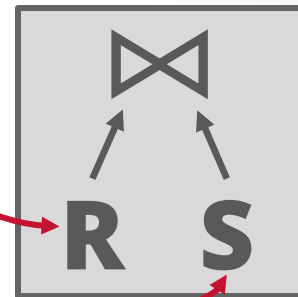
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NAÏVE NESTED LOOP JOIN



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NAÏVE NESTED LOOP JOIN

Why is this algorithm bad?

→ For every tuple in **R**, it scans **S** once

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S(id, value, cdate)

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200	8888	2/24/25

NAÏVE NESTED LOOP JOIN

Why is this algorithm bad?

→ For every tuple in **R**, it scans **S** once

Cost: $M + (m \cdot N)$

M pages
m tuples

R(id, name)

id	name
600	MethodMan
200	GZA
100	Andy
300	ODB
500	RZA
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400	Raekwon

S(id, value, cdate)

id	value	cdate
100	2222	2/24/25
500	7777	2/24/25
400	6666	2/24/25
100	9999	2/24/25
200	8888	2/24/25

N pages
n tuples

NAÏVE NESTED LOOP JOIN

Example database:

→ Table **R**: $M = 1000$, $m = 100,000$

→ Table **S**: $N = 500$, $n = 40,000$

Cost Analysis:

→ $M + (m \cdot N) = 1000 + (100000 \cdot 500) = 50,001,000$ IOs

→ At 0.1 ms/IO, Total time ≈ 1.3 hours

NAÏVE NESTED LOOP JOIN

Example database:

→ Table **R**: $M = 1000$, $m = 100,000$

→ Table **S**: $N = 500$, $n = 40,000$

Cost Analysis:

→ $M + (m \cdot N) = 1000 + (100000 \cdot 500) = 50,001,000$ IOs

→ At 0.1 ms/IO, Total time \approx 1.3 hours

What if smaller table (**S**) is used as the outer table?

→ $N + (n \cdot M) = 500 + (40000 \cdot 1000) = 40,000,500$ IOs

→ At 0.1 ms/IO, Total time \approx 1.1 hours

NAÏVE NESTED LOOP JOIN

Example database:

→ Table **R**: $M = 1000$, $m = 100,000$
 → Table **S**: $N = 500$, $n = 40,000$ } *4 KB pages → 6 MB*

Cost Analysis:

→ $M + (m \cdot N) = 1000 + (100000 \cdot 500) = 50,001,000$ IOs

→ At 0.1 ms/IO, Total time \approx 1.3 hours

What if smaller table (**S**) is used as the outer table?

→ $N + (n \cdot M) = 500 + (40000 \cdot 1000) = 40,000,500$ IOs

→ At 0.1 ms/IO, Total time \approx 1.1 hours

BLOCK NESTED LOOP JOIN

```

foreach block  $B_R \in R$ :
  foreach block  $B_S \in S$ :
    foreach tuple  $r \in B_R$ :
      foreach tuple  $s \in B_S$ :
        if  $r$  and  $s$  match then emit
  
```

$R(id, name)$

id	name
600	MethodMan
200	GZA
100	Andy
300	ODB
500	RZA
700	Ghostface
400	Raekwon

M pages
 m tuples

$S(id, value, cdate)$

id	value	cdates
100	2222	2/24/25
500	7777	2/24/25
400	6666	2/24/25
100	9999	2/24/25
200	8888	2/24/25

N pages
 n tuples

BLOCK NESTED LOOP JOIN

This algorithm performs fewer disk accesses.

→ For every block in **R**, it scans **S** once.

Cost: $M + (M \cdot N)$

M pages
 m tuples

R(id, name)

id	name
600	MethodMan
200	GZA
100	Andy
300	ODB
500	RZA
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400	Raekwon

S(id, value, cdate)

id	value	cdate
100	2222	2/24/25
500	7777	2/24/25
400	6666	2/24/25
100	9999	2/24/25
200	8888	2/24/25

N pages
 n tuples

BLOCK NESTED LOOP JOIN

The smaller table should be the outer table.

We determine size based on the number of pages,
not the number of tuples.

M pages
m tuples

R(id, name)

id	name
600	MethodMan
200	GZA
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300	ODB
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S(id, value, cdate)

id	value	cdate
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500	7777	2/24/25
400	6666	2/24/25
100	9999	2/24/25
200	8888	2/24/25

N pages
n tuples

BLOCK NESTED LOOP JOIN

If we have B buffers available:

- Use $B-2$ buffers for each block of the outer table.
- Use one buffer for the inner table, one buffer for output.

M pages
 m tuples

R(id, name)

id	name
600	MethodMan
200	GZA
100	Andy
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S(id, value, cdate)

id	value	cdates
100	2222	2/24/25
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400	6666	2/24/25
100	9999	2/24/25
200	8888	2/24/25

N pages
 n tuples

BLOCK NESTED LOOP JOIN

```

foreach  $B - 2$  pages  $p_R \in R$ :
  foreach page  $p_S \in S$ :
    foreach tuple  $r \in B - 2$  pages:
      foreach tuple  $s \in p_S$ :
        if  $r$  and  $s$  match then emit
  
```

$R(id, name)$

id	name
600	MethodMan
200	GZA
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M pages
 m tuples

$S(id, value, cdate)$

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100	9999	2/24/25
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N pages
 n tuples

BLOCK NESTED LOOP JOIN

This algorithm uses $B-2$ buffers for scanning R .

Cost: $M + (\lceil M / (B-2) \rceil \cdot N)$

If the outer relation fits in memory ($M < B-2$):

→ **Cost:** $M + N = 1000 + 500 = 1500$ I/Os

→ At 0.1ms per I/O, Total time ≈ 0.15 seconds

If we have $B=102$ buffer pages:

→ **Cost:** $M + (\lceil M / (B-2) \rceil \cdot N) = 1000 + 10 \cdot 500 = 6000$ I/Os

→ Or can switch inner/outer relations, giving us cost:

$500 + 5 \cdot 1000 = 5500$ I/Os

NESTED LOOP JOIN

Why is the basic nested loop join so bad?

→ For each tuple in the outer table, we must do a sequential scan to check for a match in the inner table.

We can avoid sequential scans by using an index to find inner table matches.

→ Use an existing index for the join.

INDEX NESTED LOOP JOIN

```

foreach tuple  $r \in R$ :
  foreach tuple  $s \in \text{Index}(r_i = s_j)$ :
    if  $r$  and  $s$  match then emit
  
```

M pages
 m tuples

$R(\text{id}, \text{name})$

id	name
600	MethodMan
200	GZA
100	Andy
300	ODB
500	RZA
700	Ghostface
400	Raekwon

$S(\text{id}, \text{value}, \text{cdate})$

id	value	cdate
100	2222	2/24/25
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400	6666	2/24/25
100	9999	2/24/25
200	8888	2/24/25

$\text{Index}(S.\text{id})$



N pages
 n tuples

INDEX NESTED LOOP JOIN

Assume the cost of each index probe is some constant C per tuple.

Cost: $M + (m \cdot C)$

M pages
 m tuples

R(id, name)

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600	MethodMan
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S(id, value, cdate)

id	value	cdate
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Index(S.id)



N pages
 n tuples

NESTED LOOP JOIN SUMMARY

Key Takeaways

- Pick the smaller table as the outer table, when possible.
- Buffer as much of the outer table in memory as possible.
- Loop over the inner table (or use an index).

Algorithms

- Naïve
- Block
- Index

SORT-MERGE JOIN

Phase #1: Sort

- Sort both tables on the join key(s).
- You can use any appropriate sort algorithm
- These phases are distinct from the sort/merge phases of an external merge sort, from the previous class

Phase #2: Merge

- Step through the two sorted tables with cursors and emit matching tuples.
- May need to backtrack depending on the join type.

SORT-MERGE JOIN

```
sort R, S on join keys
cursorR ← Rsorted, cursorS ← Ssorted
while cursorR and cursorS:
  if cursorR > cursorS:
    increment cursorS
  if cursorR < cursorS:
    increment cursorR
    backtrack cursorS (if necessary)
  elif cursorR and cursorS match:
    emit
    increment cursorS
```

SORT-MERGE JOIN

R(id, name)

id	name
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SORT-MERGE JOIN

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Sort!

S(id, value, cdate)

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Sort!

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SORT-MERGE JOIN

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Sort!

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Sort!

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SELECT R.id, S.cdate
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SORT-MERGE JOIN

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SORT-MERGE JOIN

R(id, name)



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S(id, value, cdate)



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SORT-MERGE JOIN

R(id, name)



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S(id, value, cdate)




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Last Value: ----

```
SELECT R.id, S.cdate
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SORT-MERGE JOIN

R(id, name)



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SORT-MERGE JOIN

R(id, name)



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S(id, value, cdate)



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500	7777	2/24/25

Last Value: ---


```
SELECT R.id, S.cdate
FROM R JOIN S
      ON R.id = S.id
WHERE S.value > 100
```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Andy	100	2222	2/24/25


SORT-MERGE JOIN

R(id, name)



id	name
100	Andy
200	GZA
200	GZA
300	ODB
400	Raekwon
500	RZA
600	MethodMan
700	Ghostface

S(id, value, cdate)



id	value	cdate
100	2222	2/24/25
100	9999	2/24/25
200	8888	2/24/25
400	6666	2/24/25
500	7777	2/24/25

Last Value: ---


```
SELECT R.id, S.cdate
FROM R JOIN S
      ON R.id = S.id
WHERE S.value > 100
```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Andy	100	2222	2/24/25


SORT-MERGE JOIN

R(id, name)



id	name
100	Andy
200	GZA
200	GZA
300	ODB
400	Raekwon
500	RZA
600	MethodMan
700	Ghostface

S(id, value, cdate)



id	value	cdate
100	2222	2/24/25
100	9999	2/24/25
200	8888	2/24/25
400	6666	2/24/25
500	7777	2/24/25

Last Value: ---


```
SELECT R.id, S.cdate
FROM R JOIN S
      ON R.id = S.id
WHERE S.value > 100
```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Andy	100	2222	2/24/25
100	Andy	100	9999	2/24/25


SORT-MERGE JOIN

R(id, name)



id	name
100	Andy
200	GZA
200	GZA
300	ODB
400	Raekwon
500	RZA
600	MethodMan
700	Ghostface

S(id, value, cdate)



id	value	cdate
100	2222	2/24/25
100	9999	2/24/25
200	8888	2/24/25
400	6666	2/24/25
500	7777	2/24/25

Last Value: 100

```
SELECT R.id, S.cdate
FROM R JOIN S
      ON R.id = S.id
WHERE S.value > 100
```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Andy	100	2222	2/24/25
100	Andy	100	9999	2/24/25

SORT-MERGE JOIN

R(id, name)

id	name
100	Andy
200	GZA
200	GZA
300	ODB
400	Raekwon
500	RZA
600	MethodMan
700	Ghostface

S(id, value, cdate)

id	value	cdate
100	2222	2/24/25
100	9999	2/24/25
200	8888	2/24/25
400	6666	2/24/25
500	7777	2/24/25

Last Value: 100

```
SELECT R.id, S.cdate
FROM R JOIN S
      ON R.id = S.id
WHERE S.value > 100
```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Andy	100	2222	2/24/25
100	Andy	100	9999	2/24/25

SORT-MERGE JOIN

R(id, name)

id	name
100	Andy
200	GZA
200	GZA
300	ODB
400	Raekwon
500	RZA
600	MethodMan
700	Ghostface

S(id, value, cdate)

id	value	cdate
100	2222	2/24/25
100	9999	2/24/25
200	8888	2/24/25
400	6666	2/24/25
500	7777	2/24/25

Last Value: 100

```
SELECT R.id, S.cdate
FROM R JOIN S
      ON R.id = S.id
WHERE S.value > 100
```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Andy	100	2222	2/24/25
100	Andy	100	9999	2/24/25
200	GZA	200	8888	2/24/25

SORT-MERGE JOIN

R(id, name)

id	name
100	Andy
200	GZA
200	GZA
300	ODB
400	Raekwon
500	RZA
600	MethodMan
700	Ghostface

S(id, value, cdate)

id	value	cdate
100	2222	2/24/25
100	9999	2/24/25
200	8888	2/24/25
400	6666	2/24/25
500	7777	2/24/25

Last Value: 200

```
SELECT R.id, S.cdate
FROM R JOIN S
      ON R.id = S.id
WHERE S.value > 100
```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Andy	100	2222	2/24/25
100	Andy	100	9999	2/24/25
200	GZA	200	8888	2/24/25

SORT-MERGE JOIN

R(id, name)

id	name
100	Andy
200	GZA
200	GZA
300	ODB
400	Raekwon
500	RZA
600	MethodMan
700	Ghostface

S(id, value, cdate)

id	value	cdate
100	2222	2/24/25
100	9999	2/24/25
200	8888	2/24/25
400	6666	2/24/25
500	7777	2/24/25

Last Value: 200

```
SELECT R.id, S.cdate
FROM R JOIN S
      ON R.id = S.id
WHERE S.value > 100
```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Andy	100	2222	2/24/25
100	Andy	100	9999	2/24/25
200	GZA	200	8888	2/24/25

SORT-MERGE JOIN

R(id, name)

id	name
100	Andy
200	GZA
200	GZA
300	ODB
400	Raekwon
500	RZA
600	MethodMan
700	Ghostface

S(id, value, cdate)

id	value	cdate
100	2222	2/24/25
100	9999	2/24/25
200	8888	2/24/25
400	6666	2/24/25
500	7777	2/24/25

Last Value: 200

```
SELECT R.id, S.cdate
FROM R JOIN S
      ON R.id = S.id
WHERE S.value > 100
```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Andy	100	2222	2/24/25
100	Andy	100	9999	2/24/25
200	GZA	200	8888	2/24/25

SORT-MERGE JOIN

R(id, name)

id	name
100	Andy
200	GZA
200	GZA
300	ODB
400	Raekwon
500	RZA
600	MethodMan
700	Ghostface

S(id, value, cdate)

id	value	cdate
100	2222	2/24/25
100	9999	2/24/25
200	8888	2/24/25
400	6666	2/24/25
500	7777	2/24/25

Last Value: 200

```
SELECT R.id, S.cdate
FROM R JOIN S
      ON R.id = S.id
WHERE S.value > 100
```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Andy	100	2222	2/24/25
100	Andy	100	9999	2/24/25
200	GZA	200	8888	2/24/25
200	GZA	200	8888	2/24/25

SORT-MERGE JOIN

R(id, name)

id	name
100	Andy
200	GZA
200	GZA
300	ODB
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500	RZA
600	MethodMan
700	Ghostface

S(id, value, cdate)

id	value	cdate
100	2222	2/24/25
100	9999	2/24/25
200	8888	2/24/25
400	6666	2/24/25
500	7777	2/24/25

Last Value: 200

```
SELECT R.id, S.cdate
FROM R JOIN S
      ON R.id = S.id
WHERE S.value > 100
```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Andy	100	2222	2/24/25
100	Andy	100	9999	2/24/25
200	GZA	200	8888	2/24/25
200	GZA	200	8888	2/24/25

SORT-MERGE JOIN

R(id, name)

id	name
100	Andy
200	GZA
200	GZA
300	ODB
400	Raekwon
500	RZA
600	MethodMan
700	Ghostface

S(id, value, cdate)

id	value	cdate
100	2222	2/24/25
100	9999	2/24/25
200	8888	2/24/25
400	6666	2/24/25
500	7777	2/24/25

Last Value: 200

```
SELECT R.id, S.cdate
FROM R JOIN S
      ON R.id = S.id
WHERE S.value > 100
```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Andy	100	2222	2/24/25
100	Andy	100	9999	2/24/25
200	GZA	200	8888	2/24/25
200	GZA	200	8888	2/24/25

SORT-MERGE JOIN

R(id, name)

id	name
100	Andy
200	GZA
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500	RZA
600	MethodMan
700	Ghostface

S(id, value, cdate)

id	value	cdate
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100	9999	2/24/25
200	8888	2/24/25
400	6666	2/24/25
500	7777	2/24/25

Last Value: 200

```
SELECT R.id, S.cdate
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```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Andy	100	2222	2/24/25
100	Andy	100	9999	2/24/25
200	GZA	200	8888	2/24/25
200	GZA	200	8888	2/24/25
400	Raekwon	200	6666	2/24/25

SORT-MERGE JOIN

R(id, name)

id	name
100	Andy
200	GZA
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400	Raekwon
500	RZA
600	MethodMan
700	Ghostface

S(id, value, cdate)

id	value	cdate
100	2222	2/24/25
100	9999	2/24/25
200	8888	2/24/25
400	6666	2/24/25
500	7777	2/24/25

Last Value: 400

```
SELECT R.id, S.cdate
FROM R JOIN S
      ON R.id = S.id
WHERE S.value > 100
```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Andy	100	2222	2/24/25
100	Andy	100	9999	2/24/25
200	GZA	200	8888	2/24/25
200	GZA	200	8888	2/24/25
400	Raekwon	200	6666	2/24/25

SORT-MERGE JOIN

R(id, name)

id	name
100	Andy
200	GZA
200	GZA
300	ODB
400	Raekwon
500	RZA
600	MethodMan
700	Ghostface

S(id, value, cdate)

id	value	cdate
100	2222	2/24/25
100	9999	2/24/25
200	8888	2/24/25
400	6666	2/24/25
500	7777	2/24/25

Last Value: 400

```
SELECT R.id, S.cdate
FROM R JOIN S
      ON R.id = S.id
WHERE S.value > 100
```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Andy	100	2222	2/24/25
100	Andy	100	9999	2/24/25
200	GZA	200	8888	2/24/25
200	GZA	200	8888	2/24/25
400	Raekwon	200	6666	2/24/25
500	RZA	500	7777	2/24/25

SORT-MERGE JOIN

R(id, name)

id	name
100	Andy
200	GZA
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```
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```

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R.id	R.name	S.id	S.value	S.cdate
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200	GZA	200	8888	2/24/25
200	GZA	200	8888	2/24/25
400	Raekwon	200	6666	2/24/25
500	RZA	500	7777	2/24/25

Last Value: 500

SORT-MERGE JOIN

R(id, name)

id	name
100	Andy
200	GZA
200	GZA
300	ODB
400	Raekwon
500	RZA
600	MethodMan
700	Ghostface

S(id, value, cdate)

id	value	cdate
100	2222	2/24/25
100	9999	2/24/25
200	8888	2/24/25
400	6666	2/24/25
500	7777	2/24/25

```
SELECT R.id, S.cdate
FROM R JOIN S
      ON R.id = S.id
WHERE S.value > 100
```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Andy	100	2222	2/24/25
100	Andy	100	9999	2/24/25
200	GZA	200	8888	2/24/25
200	GZA	200	8888	2/24/25
400	Raekwon	200	6666	2/24/25
500	RZA	500	7777	2/24/25

Last Value: 500

SORT-MERGE JOIN

R(id, name)

id	name
100	Andy
200	GZA
200	GZA
300	ODB
400	Raekwon
500	RZA
600	MethodMan
700	Ghostface

S(id, value, cdate)

id	value	cdate
100	2222	2/24/25
100	9999	2/24/25
200	8888	2/24/25
400	6666	2/24/25
500	7777	2/24/25

Last Value: 500

```
SELECT R.id, S.cdate
FROM R JOIN S
      ON R.id = S.id
WHERE S.value > 100
```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Andy	100	2222	2/24/25
100	Andy	100	9999	2/24/25
200	GZA	200	8888	2/24/25
200	GZA	200	8888	2/24/25
400	Raekwon	200	6666	2/24/25
500	RZA	500	7777	2/24/25

SORT-MERGE JOIN

R(id, name)

id	name
100	Andy
200	GZA
200	GZA
300	ODB
400	Raekwon
500	RZA
600	MethodMan
700	Ghostface

S(id, value, cdate)

id	value	cdate
100	2222	2/24/25
100	9999	2/24/25
200	8888	2/24/25
400	6666	2/24/25
500	7777	2/24/25

Last Value: 500

```
SELECT R.id, S.cdate
FROM R JOIN S
      ON R.id = S.id
WHERE S.value > 100
```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Andy	100	2222	2/24/25
100	Andy	100	9999	2/24/25
200	GZA	200	8888	2/24/25
200	GZA	200	8888	2/24/25
400	Raekwon	200	6666	2/24/25
500	RZA	500	7777	2/24/25

SORT-MERGE JOIN

Sort Cost (**R**): $2M \cdot (1 + \lceil \log_{B-1} [M / B] \rceil)$

Sort Cost (**S**): $2N \cdot (1 + \lceil \log_{B-1} [N / B] \rceil)$

Merge Cost: $(M + N)$

Total Cost: Sort + Merge

SORT-MERGE JOIN

Example database:

→ Table **R**: $M = 1000$, $m = 100,000$

→ Table **S**: $N = 500$, $n = 40,000$

With $B=100$ buffer pages, both **R** and **S** can be sorted in two passes:

→ Sort Cost (**R**) = $2000 \cdot (1 + \lceil \log_{99} 1000 / 100 \rceil) = 4000$ I/Os

→ Sort Cost (**S**) = $1000 \cdot (1 + \lceil \log_{99} 500 / 100 \rceil) = 2000$ I/Os

→ Merge Cost = $(1000 + 500) = 1500$ I/Os

→ Total Cost = $4000 + 2000 + 1500 = 7500$ I/Os

→ At 0.1 ms/IO, Total time ≈ 0.75 seconds

SORT-MERGE JOIN

The worst case for the merging phase is when the join attribute of all the tuples in both relations contains the same value.

Cost: $(M \cdot N) + (\text{sort cost})$

WHEN IS SORT-MERGE JOIN USEFUL?

One or both tables are already sorted on join key.

Output must be sorted on join key.

The input relations may be sorted either by an explicit sort operator, or by scanning the relation using an index on the join key.

HASH JOIN

If tuple $\mathbf{r} \in \mathbf{R}$ and tuple $\mathbf{s} \in \mathbf{S}$ satisfy the join condition, then they have the same value for the join attributes.

If that value is hashed to some partition \mathbf{i} , the \mathbf{R} tuple must be in \mathbf{r}_i and the \mathbf{S} tuple in \mathbf{s}_i .

Therefore, \mathbf{R} tuples in \mathbf{r}_i need only to be compared with \mathbf{S} tuples in \mathbf{s}_i .

SIMPLE HASH JOIN ALGORITHM

Phase #1: Build

- Scan the outer relation and populate a hash table using the hash function h_1 on the join attributes.
- We can use any hash table that we discussed before but in practice linear probing works the best.

Phase #2: Probe

- Scan the inner relation and use h_1 on each tuple to jump to a location in the hash table and find a matching tuple.

SIMPLE HASH JOIN ALGORITHM

```
foreach tuple  $r \in R$ :  
  insert  $h_1(r)$  into hash table  $HT_R$   
foreach tuple  $s \in S$ :  
  output, if  $h_1(s) \in HT_R$ 
```

$R(id, name)$

$S(id, value, cdate)$

SIMPLE HASH JOIN ALGORITHM

```

foreach tuple  $r \in R$ :
  insert  $h_1(r)$  into hash table  $HT_R$ 
foreach tuple  $s \in S$ :
  output, if  $h_1(s) \in HT_R$ 
  
```

$R(id, name)$



Hash Table

HT_R

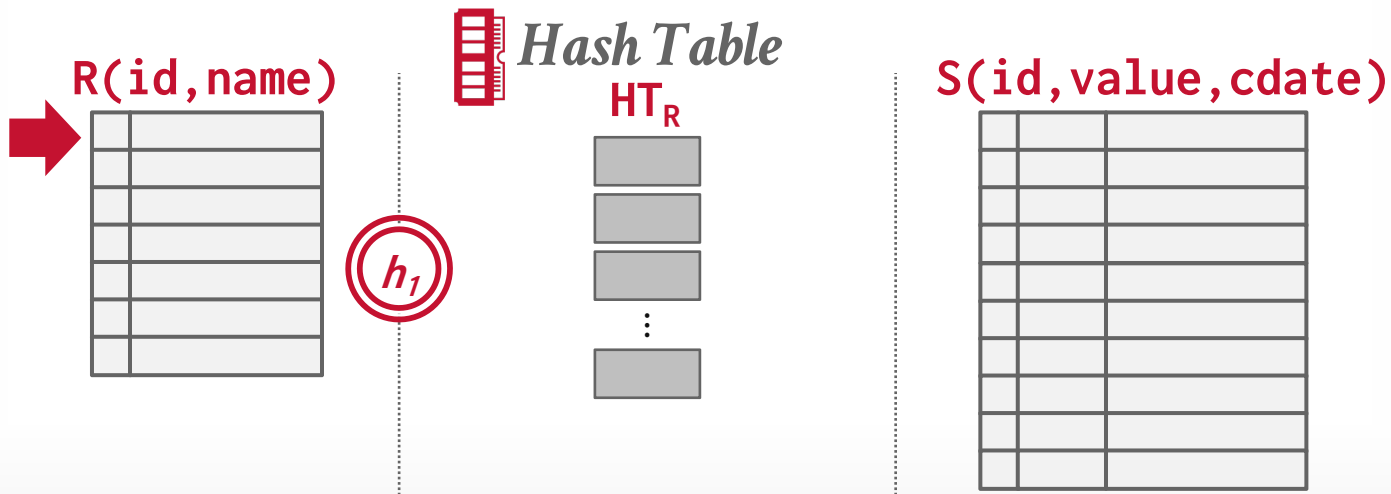
⋮

$S(id, value, cdate)$

SIMPLE HASH JOIN ALGORITHM

```

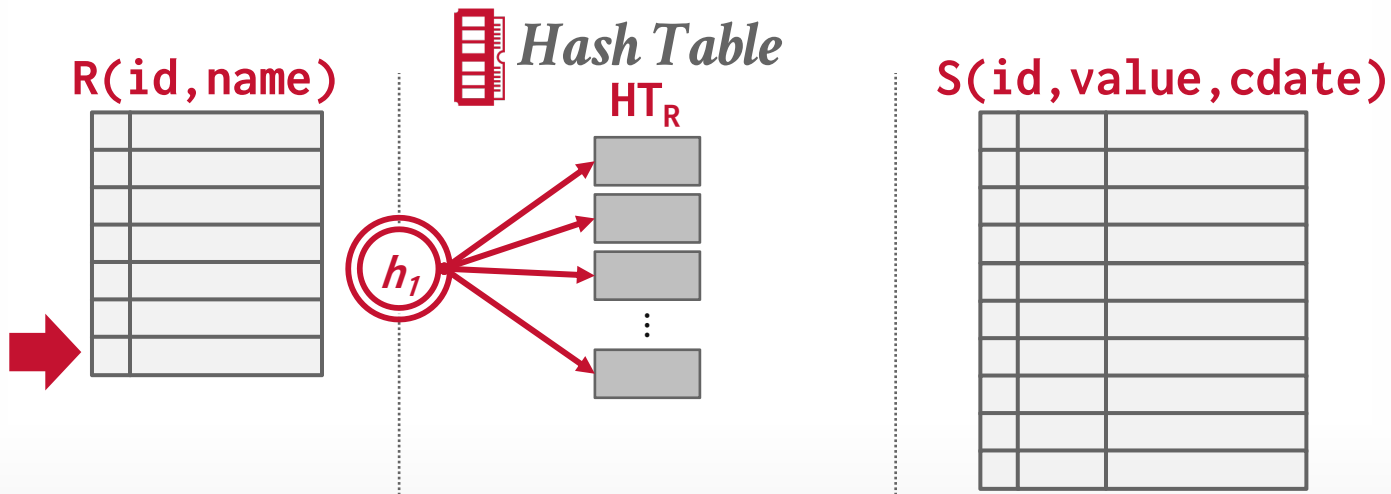
foreach tuple  $r \in R$ :
  insert  $h_1(r)$  into hash table  $HT_R$ 
foreach tuple  $s \in S$ :
  output, if  $h_1(s) \in HT_R$ 
  
```



SIMPLE HASH JOIN ALGORITHM

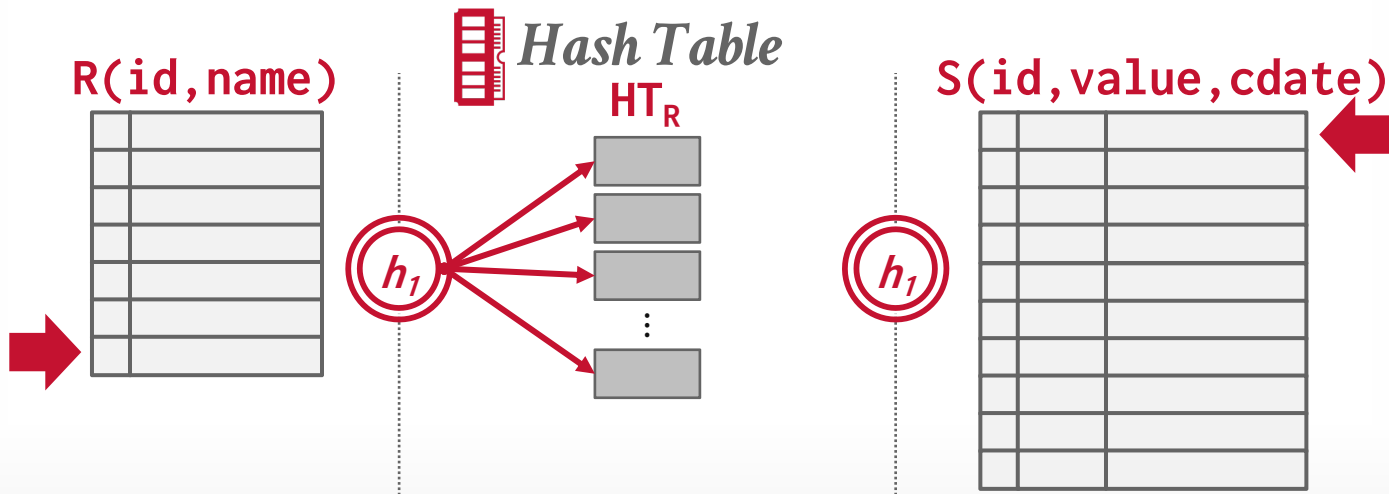
```

foreach tuple  $r \in R$ :
  insert  $h_1(r)$  into hash table  $HT_R$ 
foreach tuple  $s \in S$ :
  output, if  $h_1(s) \in HT_R$ 
  
```



SIMPLE HASH JOIN ALGORITHM

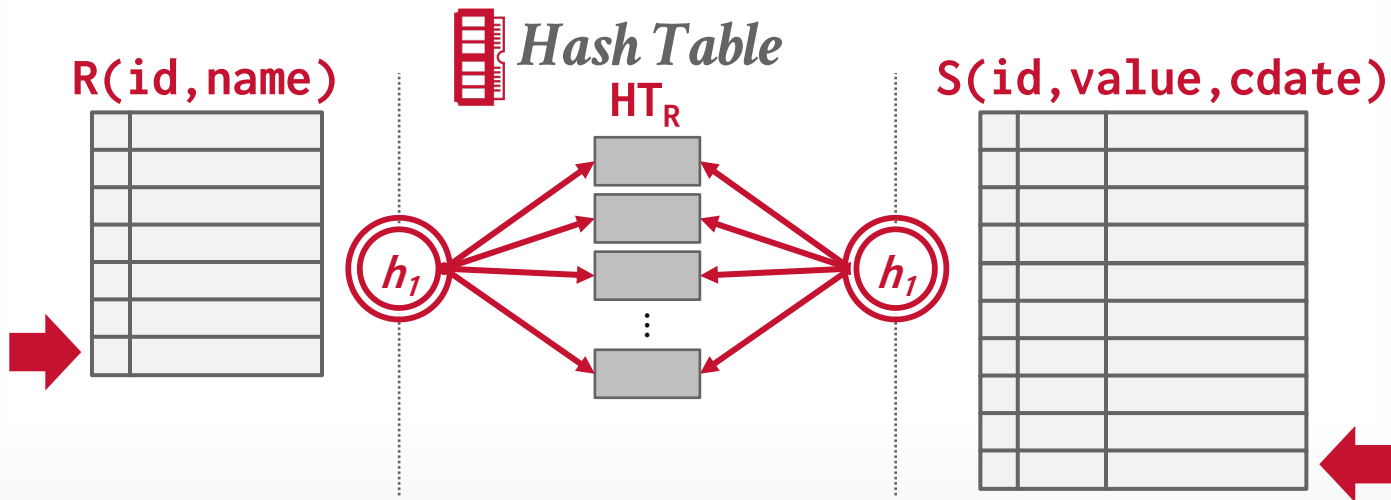
```
foreach tuple  $r \in R$ :  
  insert  $h_1(r)$  into hash table  $HT_R$   
foreach tuple  $s \in S$ :  
  output, if  $h_1(s) \in HT_R$ 
```



SIMPLE HASH JOIN ALGORITHM

```

foreach tuple  $r \in R$ :
  insert  $h_1(r)$  into hash table  $HT_R$ 
foreach tuple  $s \in S$ :
  output, if  $h_1(s) \in HT_R$ 
  
```

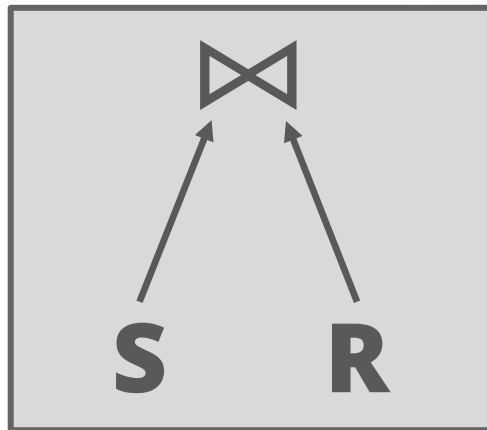


OPTIMIZATION: PROBE FILTER

Create a probe filter (Bloom Filter) as the DBMS constructs the hash table on the “build” table in the first phase.

- Always check the filter before probing the hash table.
- Faster than probing hash table because the filter fits in CPU cache.

This technique is sometimes called *sideways information passing*.

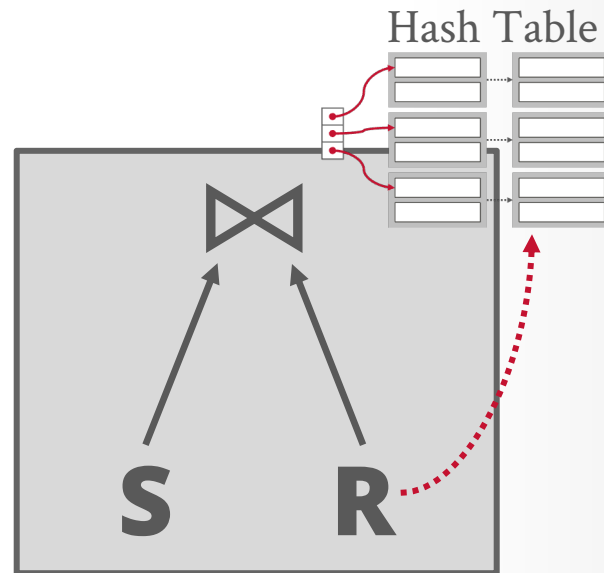


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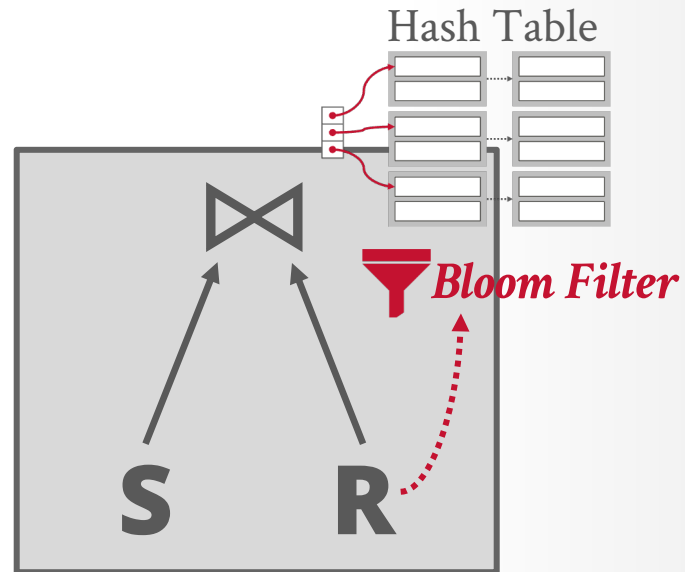


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- Faster than probing hash table because the filter fits in CPU cache.

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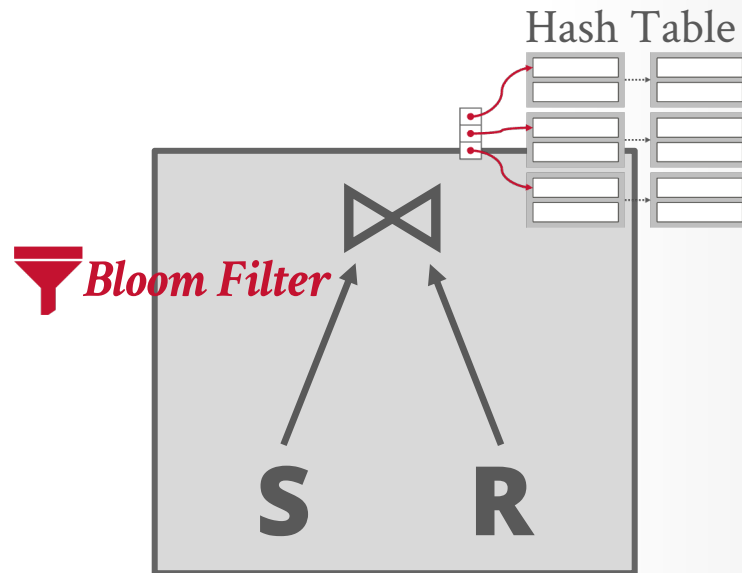


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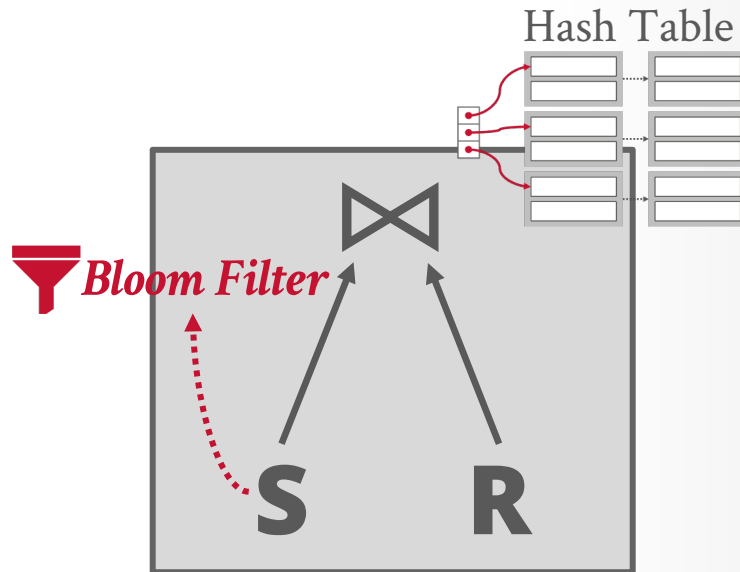


OPTIMIZATION: PROBE FILTER

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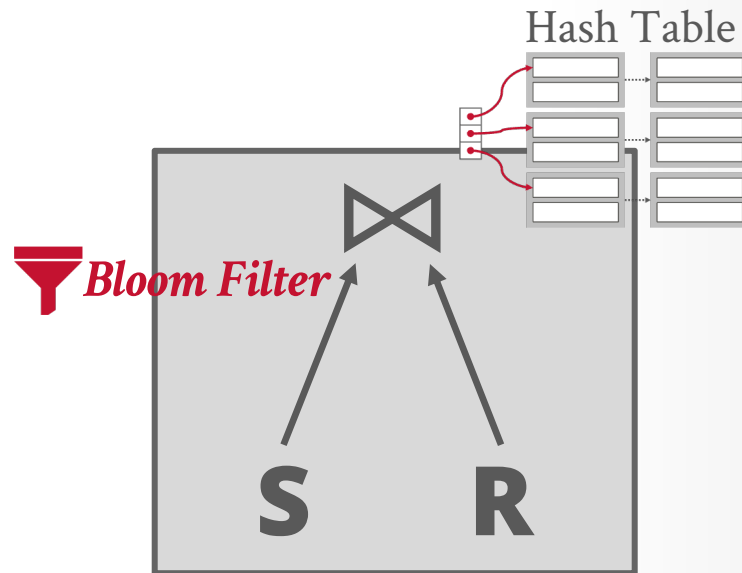


OPTIMIZATION: PROBE FILTER

Create a probe filter (Bloom Filter) as the DBMS constructs the hash table on the “build” table in the first phase.

- Always check the filter before probing the hash table.
- Faster than probing hash table because the filter fits in CPU cache.

This technique is sometimes called *sideways information passing*.



HASH JOINS OF LARGE RELATIONS

What happens if we do not have enough memory to fit the entire hash table?

We do not want to let the buffer pool manager swap out the hash table pages at random.

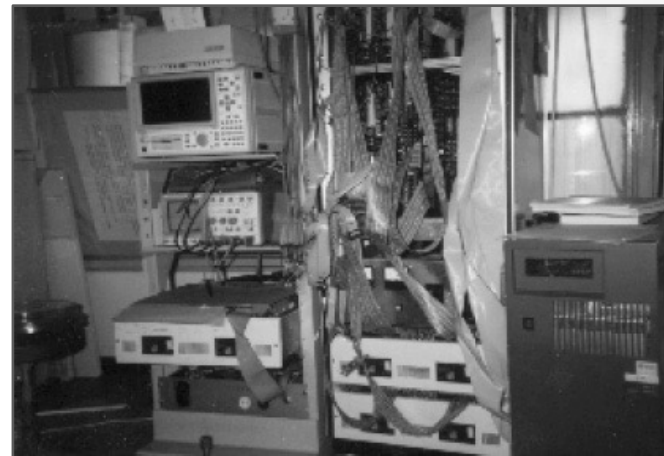
PARTITIONED HASH JOIN

Hash join when tables do not fit in memory.

- **Partition Phase:** Hash both tables on the join attribute into partitions.
- **Probe Phase:** Compares tuples in corresponding partitions for each table.

Sometimes called **GRACE Hash Join**.

- Named after the GRACE database machine from Japan in the 1980s.



GRACE
University of Tokyo

Britton-Lee's technical achievements have created the Intelligent Data Base Machine, oriented to managers who know the value of a responsive information system. Truly user-oriented—even to

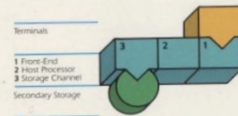
people without programming knowledge—the IDM 500 provides some remarkable advantages. Imagine how the features described inside can improve YOUR company's information productivity...

NOW

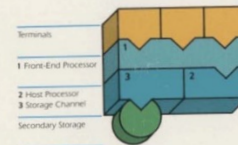


The IDM 500 A Logical Development

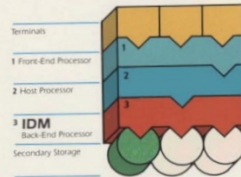
As data systems have evolved, the presence of special-purpose elements has become increasingly important, as these diagrams will illustrate:



In the 1960's, a single central processing unit (CPU) was required to monitor time-sharing among terminal users; to batch process computing tasks, and to control the access to stored data.



Through the development of front-end communication processors, the workload on the CPU was reduced. It was then able to perform its basic task of data processing much more efficiently. But the task of managing the data base was still imposed upon it.



Now Britton-Lee's IDM 500 special-purpose, back-end data-base processor brings full efficiency to the host computer and intelligent terminals, so that they can properly perform their correct functions.



Choosing the best fit Key indicators



IBM Netezza

- Performance and Price/performance leader
- Speed and ease of deployment and administration

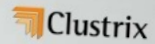
IBM Netezza standalone appliance

- Strategic requirement for standalone decision support system
- If primary data feeds are from distributed applications
- Deep analytics applications or in-database mining

IBM DB2 Analytics Accelerator for z/OS

- Transparent acceleration of existing reporting workload on DB2

CLUSTRIX APPLIANCE



Clustrix Appliance 3 Node Cluster (CLX 4110)

- 24 Intel Xeon CPU cores
- 144GB RAM
- 6GB NVRAM
- 1.35TB Intel SSD protected (2.7TB raw) data capacity

Teradata IntelliFlex™ 100% Solid State Performance

Up to: **7.5x** Performance for Com Intensive Analytics

4.5x Performance for Data Warehouse Analytics

3.5x Data Capacity

2.0x Performance per kW



Note: comparisons to the previous generation IntelliFlex platform are on a per cabinet basis. Workloads will see up to this amount of benefit.

Complete Family Of Database Machines For OLTP, Data Warehousing & Consolidated Workloads

Oracle Exadata X2-2



- Quarter, Half, Full and Multi-Racks

Oracle Exadata X2-8



- Full and Multi-Racks

Choosing the best fit Key indicators



Yellowbrick Data Warehouse Architecture

Real-time Feeds
Ingest IoT or OLTP data
Capture 100,000s
of rows per second



Periodic Bulk Loads
Capture terabytes
of data, petabytes
over time



Load and Transform
Use existing ETL tools including
intensive push-down ELT



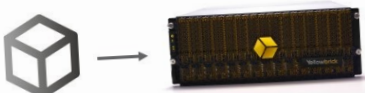
Interactive Applications
Serve short queries in
under 100 milliseconds



Powerful Analytics
Respond to
complex BI queries
in just a few seconds



Business Critical Reporting
Workload management
for prioritized responses



Source: yellowbrickdata.com

Teradata IntelliFlex 100% Solid State Performance

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de Cluster (CLX 4110)

Database Machines & Consolidated Workloads

Oracle Exadata X2-8



• Quarter, Half, Full and Multi-Racks

• Full and Multi-Racks

PARTITIONED HASH JOIN PARTITION PHASE

Hash **R** into k buckets.

Hash **S** into k buckets with same hash function.

Write buckets to disk when they get full.

R(id, name)

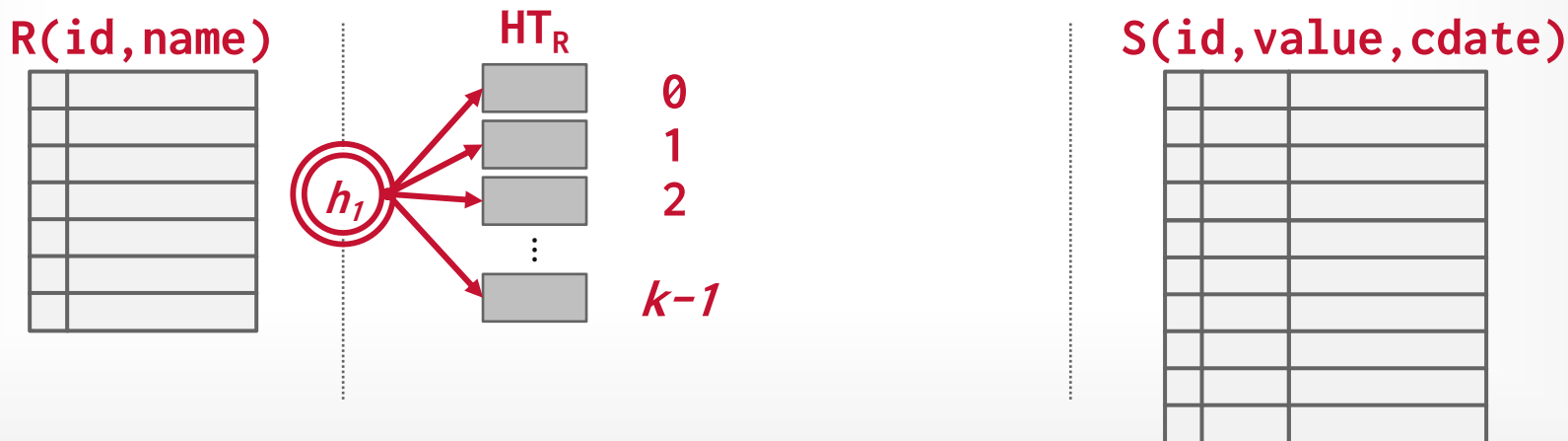
S(id, value, cdate)

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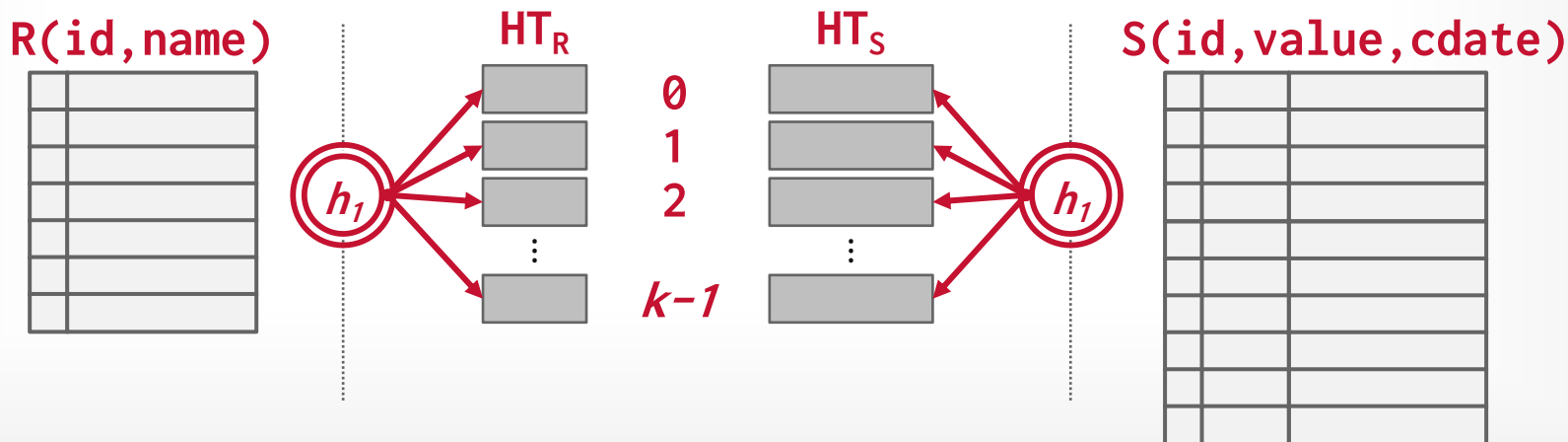


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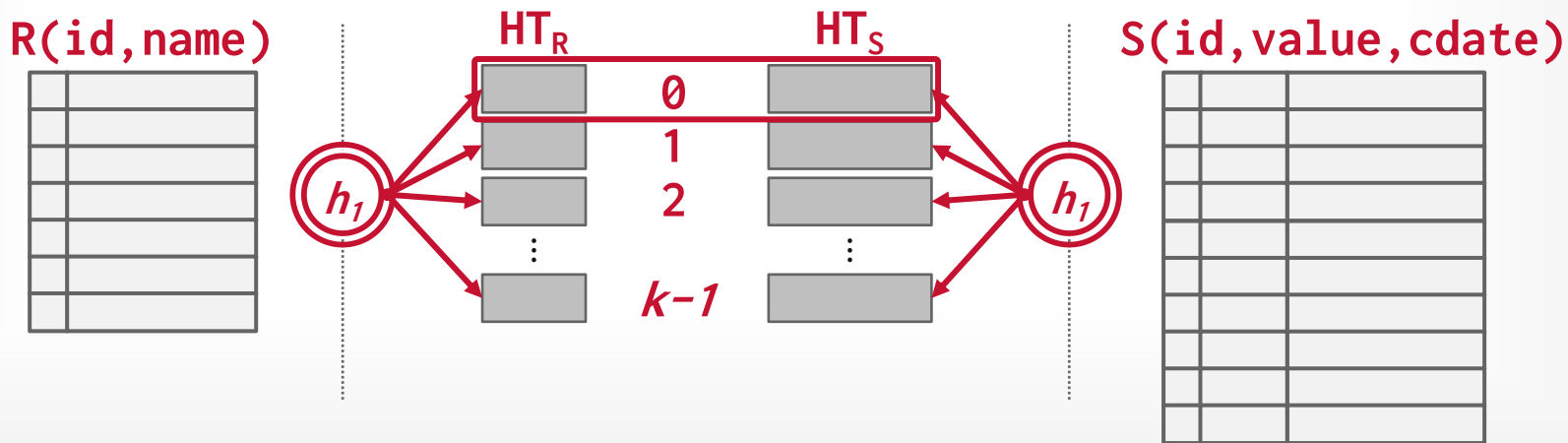
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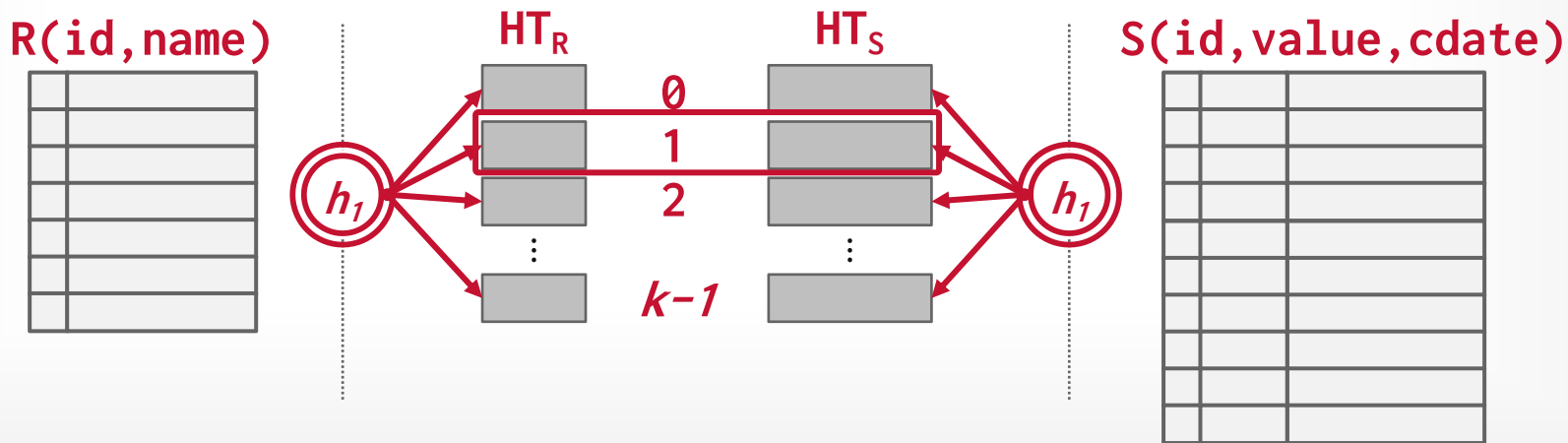
PARTITIONED HASH JOIN PROBE PHASE

Read corresponding partitions into memory one pair at a time, hash join their contents.



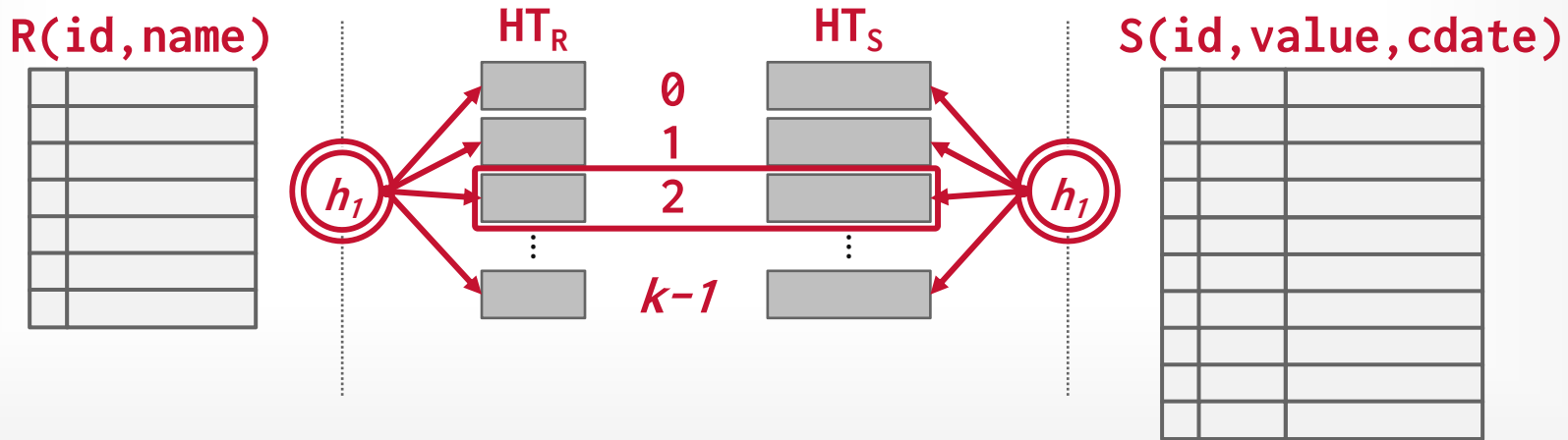
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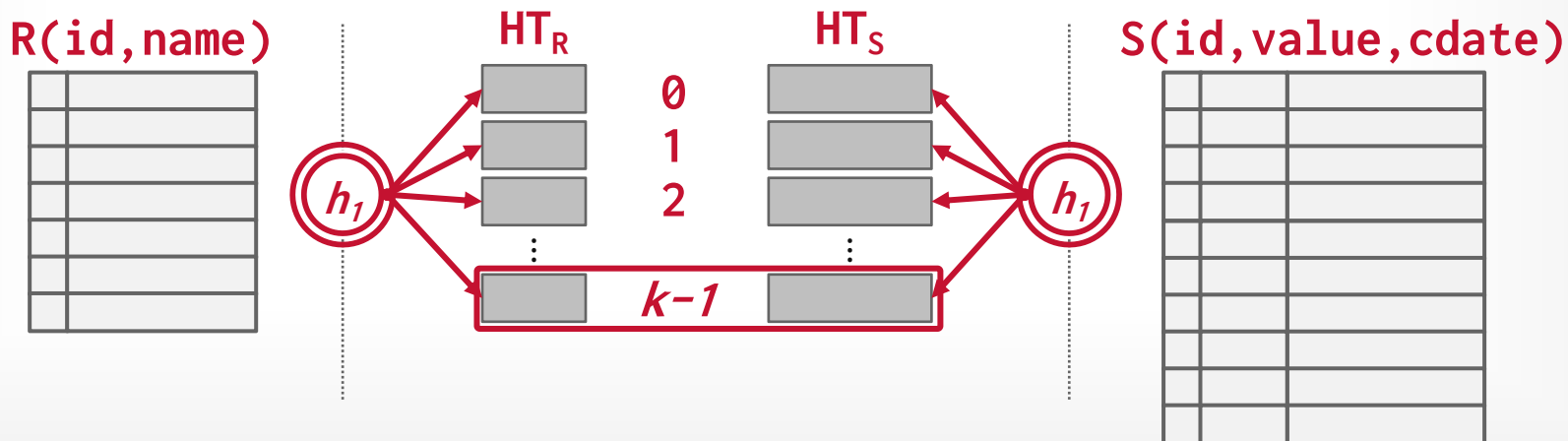
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PARTITIONED HASH JOIN EDGE CASES

If a partition does not fit in memory, recursively partition it with a different hash function

→ Repeat as needed

→ Eventually hash join the corresponding (sub-)partitions

If a single join key has too many matching records that do not fit in memory, use a **block nested loop** join just for that key.

→ Avoids random I/O in exchange for sequential I/O.

RECURSIVE PARTITIONING

$R(\text{id}, \text{name})$

h_1

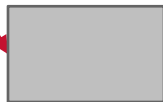


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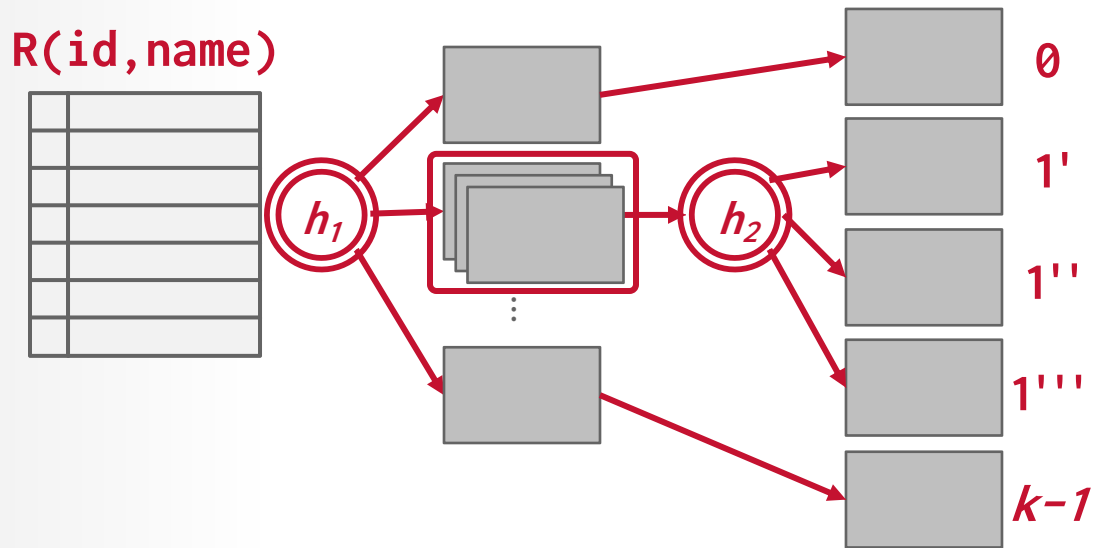
1

\vdots

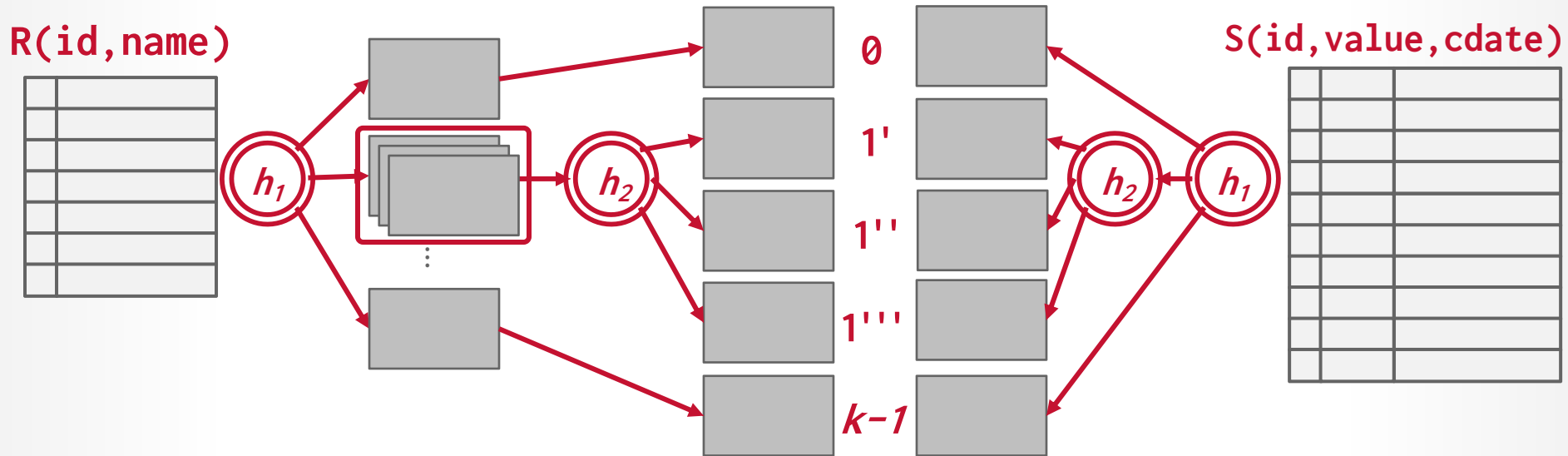


$k-1$

RECURSIVE PARTITIONING



RECURSIVE PARTITIONING



COST OF PARTITIONED HASH JOIN

If we do not need recursive partitioning:

→ Cost: $3(M + N)$

Partition phase:

→ Read+write both tables

→ $2(M+N)$ I/Os

Probe phase:

→ Read both tables (in total, one partition at a time)

→ $M+N$ I/Os

PARTITIONED HASH JOIN

Example database:

→ $M = 1000$, $m = 100,000$

→ $N = 500$, $n = 40,000$

Cost Analysis:

→ $3(M + N) = 3 \cdot (1000 + 500) = 4,500$ IOs

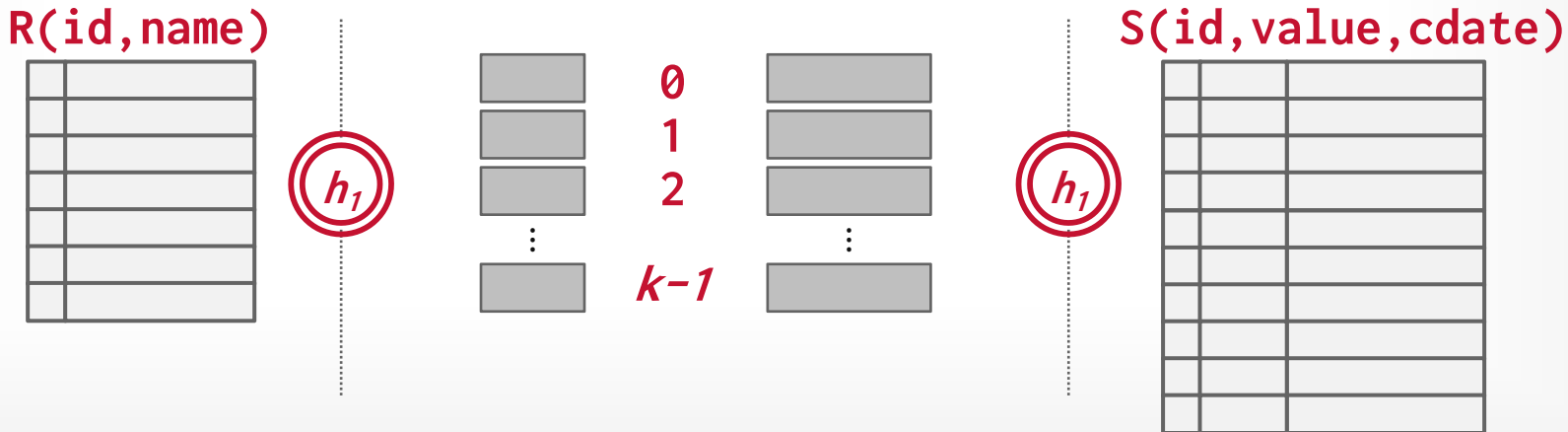
→ At 0.1 ms/IO, Total time ≈ 0.45 seconds

OPTIMIZATION: HYBRID HASH JOIN

If the buffer pool is much larger than k , then use the “extra” pages to build the hash table for the first partition in memory and join it, avoiding spilling it to disk.

→ Difficult to get to work correctly. Rarely done in practice.

→ Better to use the buffer pool pages to over partition.

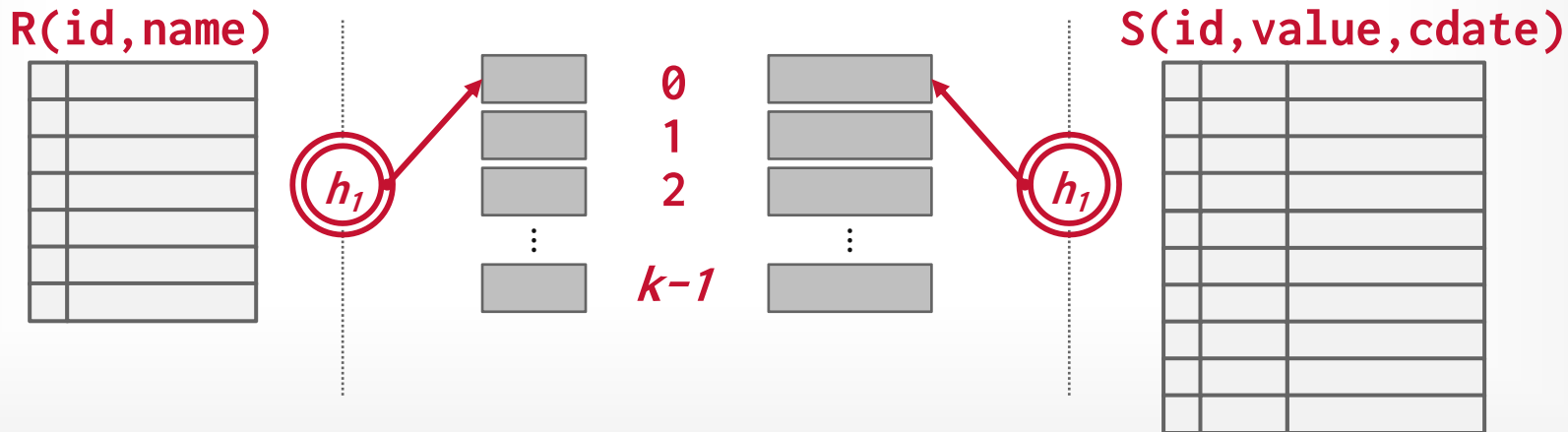


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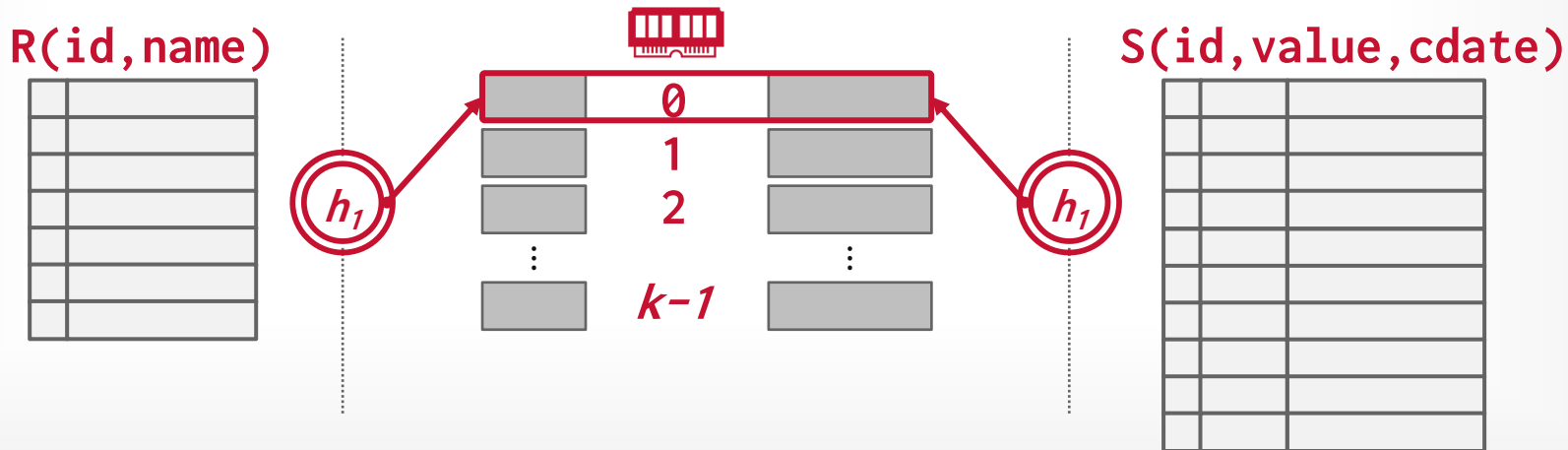


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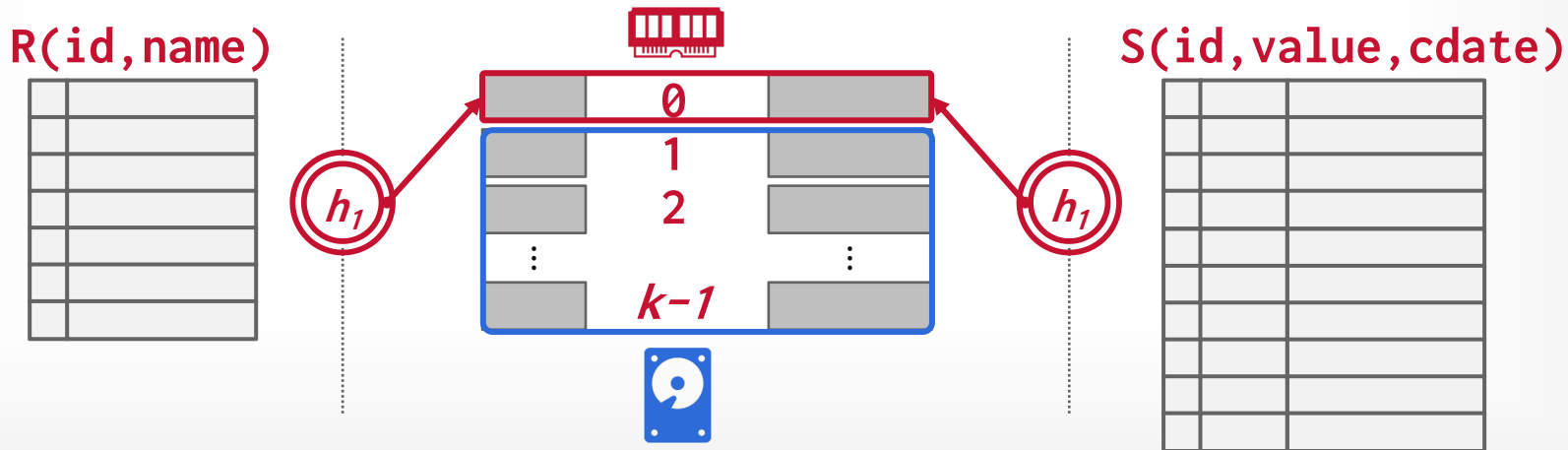


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HASH JOIN OBSERVATIONS

The probe table can be any size.

→ Only the build table (or its partitions) need to fit in memory

If the probe table entirely fits in memory, then simply build the hash table on the probe side, and scan the build side probing the hash table (no partitioning needed).

If we do not know the size, then we must use a dynamic hash table or allow for overflow pages.

JOIN ALGORITHMS: SUMMARY

Algorithm	IO Cost	Example
Naïve Nested Loop Join	$M + (m \cdot N)$	1.3 hours
Block Nested Loop Join	$M + (\lceil M / (B-2) \rceil \cdot N)$	0.55 seconds
Index Nested Loop Join	$M + (m \cdot C)$	Variable
Sort-Merge Join	$M + N + (\text{sort cost})$	0.75 seconds
Hash Join	$3 \cdot (M + N)$	0.45 seconds

CONCLUSION

Hashing is almost always better than sorting for operator execution.

Caveats:

- Sorting is better on non-uniform data.
- Sorting is better when result needs to be sorted.

Good DBMSs use either (or both).

NEXT CLASS

Mid-Term Exam!